DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

THE ANALYSIS OF OBSERVATIONAL DATA FROM NATURAL BEACHES

TECHNICAL MEMORANDUM NO. 130

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FOREWORD

An extensive literature has accumulated on natural beach data and on the results of wave-tank experiments; and current interest in both subjects is very high among geologists, oceanographers, and engineers. Many fundamental relations between wave characteristics and the behavior of beaches were obtained from wave-tank data, where conditions are controlled; whereas similar observations on natural beaches have in the past been hindered to some extent by the numerous complexly interlocked variables that operate simultaneously in nature.

The advent of high-speed computers, and the development of statistical and mathematical methods for handling large and complex sets of data have opened new doors for the analysis of natural beach data. Studies designed to take advantage of these newer techniques need to be carefully designed, however, so that the many sources of variability in the data can be "tagged" and separated from the underlying main effects that are of major importance. Without such design the data may contain much "unexplained" residual variability that cannot be distinguished from "noise".

This Technical Memorandum has been designed in part to set the newer approaches toward natural beach studies in a framework that shows the relation between wave-tank data and natural beach data. Certain underlying models, conceptual, physical, and statistical, that apply in these two cases are discussed and in part illustrated. At the time this study was organized, in 1957, only limited amount of data for illustration were available, and these had been collected for another purpose at Mission Beach, California. These data, though well-designed for their original use, also serve as an illustration of relatively "noisy data" in the present context, and they are used to illustrate one approach to the analysis of sets of natural data containing several simultaneously varying elements.

Much more important for present objectives are some generalizations that can be derived from the Mission Beach data regarding the design of natural beach studies. It is evident that proper design for fundamental studies of shore processes and beach responses needs to consider the objectives of the study, the selection of a physical or statistical model, and the collection of observational data in accordance with a plan that permits extraction of a maximum amount of information from the measurements. The Mission Bay data clearly show, for example, that a time-lag is present between initiation of a given process element (such as a change in wave height) and a response element (such as foreshore slope). This supports what has long been recognized in wave-tank experiments, and points up the need for designs that include explicit provisions for such effects.

This report was prepared by Dr. W. C. Krumbein of Northwestern University in pursuance of Contract DA-49-055 Civ Eng-62-4 with the Beach Erosion Board. At the time this report was completed, the technical staff of the Board was under the supervision of Major General Keith R. Barney, President of the Board, Colonel H. E. Sprague, Executive, and R. O. Eaton, Chief Technical Advisor. The writer is indebted to Mr. R. O. Eaton, Board's Chief Technical Advisor, and to Messrs. J. V. Hall, Jr., G. M. Watts, N. E. Taney, and J. M. Caldwell, all of whom have been very helpful in critical discussions of the concepts presented, and for reading all or parts of this report. Mr. John C. Fairchild of the Beach Erosion Board staff, compiled the Mission Beach data from Board records. Funds for computer time were made available by the Board, and additional machine time on an IBM 650 was generously provided by the Computing Center at Northwestern University, to whom the writer is also indebted for many courtesies extended.

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THE ANALYSIS OF OBSERVATIONAL DATA FROM NATURAL BEACHES

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THE GENERAL BEACH PROCESS

In its broadest sense the general beach process is a phenomenon representing the interplay between material and energy in that critical zone along coasts where wave energy is released during wave breaking. The beach process requires a physical framework of sloping sea bottom composed at least in part of movable material, such as sand or pebbles. Landward sources of similar material fed to the shore by streams, or eroded by waves, may constitute an important part of the physical framework. The interaction between energy and material in these circumstances tends to produce a linear deposit along the shore.

This beach deposit, considered as a three-dimensional body of sediment, is appropriately referred to as a landform in that it is formed by gradational agents of erosion and deposition, and it has recognizable geomorphic characteristics such as width, areal extent, and elevation with respect to sea level.

The sediments of which the beach is composed also have measurable properties, such as average particle size, degree of sorting, particle shape, and so on, that may vary from place to place on and within the beach.

Changes in the geometrical properties of the beach, or in the characteristics of its material, occur as responses to changes in beach process factors that supply energy to the beach zone. Other kinds of responses, such as changes in the amount of longshore drift, may also occur, as a result of changes in the source material supplied to the beach by landward agencies. The beach process factors, among which are wave height, wave period, and angle of wave attack, tend to control the day-to-day changes on the beach; whereas source materials and other geological factors in large part control the long-time responses in the volume, width, length, and composition of the beach deposit as a whole.

We may thus discern two broad features in the physical framework of beach analysis. The first of these, the process factors or elements, represent "independent variables" that control the physical events that occur on the beach. The second feature, the responses, represent "dependent variables" that change as the process elements themselves change in their behavior through time.

In this idealized physical sense one may think of beach process factors as causes, and beach responses as effects. However, this simple cause-and-effect viewpoint is complicated by numerous interrelations among the independent variables. These interrelations tend to obscure the underlying control-response picture by producing complex interlocking phenomena.

This study originated in 1957 when it became apparent that the increasing availability of high-speed computers, and the development of statistical methods for handling complexly interlocked sets of measurements, made it possible to re-examine the general problem of analyzing natural beach data with more certainty that the sources of variability in the data could be "sorted out" and evaluated. Moreover, the concept of specific models as frameworks of analysis was just beginning to emerge in geology, and few computer programs were available for the specific kinds of analysis contemplated.

Preliminary analysis of the data was carried out in late 1957, and the writer presented some of his findings, especially as they concerned the time lag in slope adjustment, at the Seminar on Sand Bypassing and Design of Groins held at Princeton University, October 10-12, 1958. It is hoped that this publication of the results in more formal fashion may stimulate further consideration of design elements that deserve attention in beach projects, as well as to stimulate the search for additional methods for analyzing the complex interrelations among natural beach phenomena.

WAVE TANK AND NATURAL BEACH

Interrelations among beach factors may be so complex that it is sometimes difficult to distinguish even between control and response, especially in the study of natural beaches. Because of such complexities, engineers have developed wave tank experimentation as a method for reducing the complexities to manageable proportions.

Observational data collected on natural beaches differ in several important ways from experimental data obtained in wave tank studies. In the wave tank it is possible to specify the initial state of the beach, the mean particle size of the beach material, and the direction of wave approach. Within this fixed framework beach process factors such as wave height and period may be changed systematically over some predetermined range, to observe how the beach properties and the size characteristics of the beach material respond to changing wave conditions. One of the most important aspects of wave tank experimentation is the opportunity it provides for elimination of unwanted sources of variability in the data by control of the processes that are permitted to operate.

Wave tank experiments may be characterized as including the following features: (1) a framework of analysis involving fixed initial conditions of beach material, beach slope, beach width, and angle of wave approach; (2) a set of independent variables represented by beach process factors that can be controlled at pre-determined levels, and (3) a series of dependent variables which represent responses of the beach and beach material to each operating level of the independent variables. In this way particular aspects of the general beach process may be examined as a basis for setting up analytical relations among dependent and independent variables; or to test predictions derived from theoretical considerations concerning beach processes.

Wave tank experiments generally represent scale model studies in which the physical framework of analysis is reduced to dimensions that can be practically handled. Such experiments accordingly involve an actual physical model as presented by the tank, as well as a conceptual model, either physical or mathematical, that provides a basis for organizing the observational data of the experiment.

Field observations on natural beaches require a much more complicated framework of analysis, in that process factors operate beyond man's control, they commonly vary simultaneously over a wide or narrow range, and the responses in beach properties and particle attributes depend in large part on the kinds of material already present along the shore, as well as upon natural events preceding the time of study. Moreover, some of the variables, both independent and dependent, are interrelated in complex ways that are difficult to isolate and to identify. Thus, along a particular shoreline there may be a natural relation between period and height of the waves, although in theory these may be considered as independent of each other. Such natural relations give rise to unwanted correlation effects, so that for a given study the assumed independent variables may be in practice actually interrelated. Similarly, although beach slope and average particle size on the foreshore are correctly interpreted as responses to changing wave characteristics, there is a relation between grain size and slope that appears to hold regardless of the particular wave climate in which a set of beach observations is made.

Not only are more factors involved in natural situations than in wave tanks, but there is greater difficulty in evaluating specific factors and responses on natural beaches. It is not unusual for more than one wave train to approach the beach simultaneously, each with its own period and height. This causes obvious difficulties in evaluating energy distribution on the beach. Moreover, some important process factors and responses may not be amenable to direct measurement with present methods; or they may be overlooked in the multiplicity of details that occur on natural beaches. All such variables, whether measured or not, add their degree of variability to the observational data. If important effects are omitted, the observational data may be plagued by high degrees of variability that cannot be separated from what may be called a general background of "noise".

In addition to the major process factors and beach responses that may be the main subject of study, natural phenomena are subject to a number of small scale variations, which give rise to local sporadic fluctuations in the observations. Thus, the presence of slight irregularities on the nearshore bottom of an otherwise uniform coast may produce very local changes in energy distribution, resulting in local responses that differ from the expected response in terms of the over-all conditions that are present. If observations are made in areas subject to these local effects the measurements tend, by their local variations, to add additional "noise" to the data.

The many simultaneously-varying factors in natural beaches thus operate at several time and space scales. As a result, field studies require unusually careful design to make certain that the small scale fluctuations associated with particular times or places can be identified as such, and can be separately evaluated without seriously affecting interpretation of the larger features of the beach process.

Natural beach observations, in contrast to wave tank measurements, are seen to lack most of the advantages that go with controlled experimentation. Thus, the initial state of the beach in a field study is itself the response of beach materials to processes that were going on before the study was started. On a natural beach we therefore begin with an initial state that is determined by preceding events over which we have no control. In addition, such features as the period and height of the waves, as well as their angle of approach, are not subject to direct control, and all of them may change during the course of the study. As a consequence, responses of the beach to natural conditions will in general be more complex than under controlled experimental wave tank conditions.

Although it would appear to be almost hopeless to gain reliable and meaningful quantitative generalizations from observations on natural beaches, this is far from the truth. For one thing, scale model theory is not required in observations on natural beaches, thereby removing one limitation that applies to most wave tank experiments. Moreover, within the past decade mathematical and statistical methods have been developed that make it possible to "sort out" as many sources of variation in natural data as can be designed into a given comprehensive study. The increased availability of high speed computers and automatic data processing techniques has made it feasible to analyze complex sets of natural observational data as quickly and conveniently as formerly applied to analysis of more limited sets of experimental data.

FIELD OBSERVATIONS ON BEACHES

In the study of natural beaches, as pointed out earlier, the process factors are beyond man's control and they may vary simultaneously over a considerable range. In addition, observational data usually are limited to measurement of only parts of the over-all phenomenon, inasmuch as samples of sand as well as "samples" of wave directions, periods, and heights must be depended upon as basic measurement data. Examination of the whole beach phenomenon is either physically impossible or far too costly for studies that would otherwise require measurement of every sand grain on the beach and of every wave that breaks along the shore.

Reliance upon information supplied by samples, from which inferences or predictions are made regarding the larger class of events that are of direct interest, means that the study of natural beach processes is basically statistical. Statistical analysis therefore provides a rational basis for study of natural beach phenomena in that it includes: (1) specification of a set of variables to be measured, (2) a specified plan of sampling, (3) repetition of the sampling procedure at randomized or systematically spaced times and places, (4) analysis of the sample data to "sort out" the several sources of variability in the beach process, and (5) use of the sample analysis to make predictions or generalizations about beach processes stated in terms of the likelihood that they are "true" within specified limits.

A statistical model can be set up that explicitly includes a number of process factors and beach responses occurring simultaneously at several levels of the time and space scales. Such a model provides a framework for organizing the observational data, and because it specifies a design for taking observations, the several variables may be evaluated individually in terms of their contributions even in the presence of complex relationships with other variables that change simultaneously.

Statistical models thus recognize explicitly that all factors in the general beach process add some degree of variability to the observational data. Through appropriate analysis the main factors can be separated from the more nearly random fluctuations of the small scale features in the process.

A second approach to beach studies is primarily "analytical" in the sense that a conceptual physical or mathematical model for the beach is first set up. This model specifies physical processes that occur along the beach, and it predicts responses that should arise from these physical processes. Thus, one specification may be that the wave energy in the system be largely dissipated in a relatively narrow band along the plunge zone, with wave uprush releasing part of the energy on the foreshore. From this aspect of the model we may predict that the expected response should be a linear beach deposit with a greater rate of change in attributes normal to the shoreline than in an alongshore direction. Similarly, the model may specify a strength of shore current related to the angle of wave approach, thus permitting some prediction of the component of wave energy assignable to the generation of such currents.

The conceptual physical model for the natural beach is thus much like the wave tank model, in that it takes explicit account of known or expected physical relationships between wave energy and beach material. It differs from the wave tank model in that it does not require use of scale model theory, but it does require cognizance of the large number of uncontrolled variables present and of their simultaneous variation. In this latter respect it is similar to the statistical beach model.

Beach models for field observation, whether statistical or "analytical", require selection of variables to be observed in the same way that we select process elements for wave tank studies. That is, the conceptual beach model must be comprehensive enough to specify the main operative process factors, but it may treat small-scale fluctuations as incidental components of the total "noise" in the data, to be essentially ignored during subsequent analysis. Elimination of these small-scale fluctuations may be accomplished by trend surface analysis, to be discussed more fully in a later section of this report.

An "analytical" model, by definition, is based explicitly on known or inferred physical relations between energy and material in the beach process. In statistical studies the model may also include these same physical relations among its "main factors." In addition, the statistical model pays explicit attention to sampling error which may be wholly or largely ignored in the "analytical" model. Some analytical studies, based on data "as they come" without regard for sampling problems, may thus be fully as empirical in one sense as are such statistical studies that pay attention to sampling, but seek mainly for relationships among variables without explicit regard for their underlying physical significance.

In this report the approach to natural beach studies is a general one that includes aspects of both the statistical and the conceptual physical model. That is, an attempt is made to set up an implicit function for the general beach process, from which a series of specific models may be derived, some statistical and others based directly on least squares analysis without detailed consideration of the sampling problem.

A GENERALIZED BEACH MODEL

As indicated in the previous discussion, the general beach process involves a large number of variables that may be interlocked in various ways. Because of the complex nature of the process viewed as a whole, it would seem that a broad implicit function can better serve as a generalized mathematical model than can some function that explicitly defines degrees of dependence.

In particular, it seems appropriate to group the variables concerned according to whether they represent process factors, geometrical attributes of the beach deposit, or properties of the material making up the beach. From these classes may be derived a large number of specific models that explicitly state degrees of dependence between variables in each class in terms of particular beach aspects emphasized in a given study.

A generalized implicit function of this kind is shown in the following equation:

$$F(p, G, P, S, T) = 0 \qquad \ldots (1)$$

where p represents a number of physical, chemical, and mineralogical properties of beach sediments (p_1, p_2, ...); G represents a number of geometrical properties (G_1, G_2, ...) of the beach deposit considered as a three-dimensional body of sediment; P represents individual beach or shore process factors P_1, P_2, ...; S represents geographic coordinates along and across the beach (S_1 and S_2) as well as elevation S_3 above or below sealevel; and T is a time factor. Table 1 lists a number of items in each of the first three categories of the equation, to make explicit the nature of the variables in the general beach model.

Equation (1) is a multivariate complex from which a large number of univariate or multivariate models may be derived for wave tank studies, for statistical analysis of natural beach phenomena, or for "analytical" beach models of the kind described earlier. In fact, equation (1) suggests that the general beach model can be divided into two parts, the first of which may be called the <u>process model</u>, and the second the <u>response model</u>. That is, the particle properties of the beach material and the geometric characteristics of the beach deposit are in large part responses to the action of the process elements, with modifications related to geological factors that control the mineral composition and initial textural attributes of the beach material. In a broad way equation (1) can be re-expressed as $(p,G) = f(P, S, T)$, in which the left hand side contains the response elements and the right hand side the process elements. Such re-expression is oversimplified in some ways, in that the process elements themselves are functions of the space coordinates and of time; and some aspects of the process elements, such as the pattern of wave refraction, depend upon the bottom slope, which is a geometric element on the response side of the equation.

What this sums up to is that any variable in equation (1) may be considered as either dependent or independent, in terms of the manner in which a study is designed. For this reason it was thought desirable to express the general equation in implicit form, though there are advantages in some studies to contrast process and response models directly. Equation (1) may, indeed, be expanded to include the underlying geological factors

TABLE 1

BEACH PROPERTIES AND PROCESS ELEMENTS*

A. Properties and Attributes of Beach Sediments

1. Frequency distributions of particle properties (including the mean, the standard deviation, the skewness, and the kurtosis) of the following particle properties:

   Particle size; particle sphericity (shape); particle roundness; particle orientation.

2. Mineral composition of beach particles, including the "light" and "heavy" (or magnetic) fractions, expressed as percentages.

3. Mass (aggregate) properties of the sediments, based on measurements of undisturbed samples:

   Moisture content; porosity; permeability; density (wet or dry bulk); penetrability.

4. Sedimentary structures:

   Bedding; cross-bedding; ripplemarks.

5. Amount of longshore drift.

B. Geometric Properties of the Beach as a Whole

1. Backshore width; elevation of berm; thickness of deposit.

2. Foreshore width; slope; curvature of beach in plan view.

3. Nearshore bottom depth contours.

C. Beach Process Elements or Factors

1. Waves: height; period; steepness; direction of approach; energy; breaking height; nature of surf zone; refraction patterns; velocity of wave uprush.

2. Tides: stage; time since preceding spring tide; range; periodicity.

3. Currents: wave-generated, in terms of direction and velocity; tidal currents.

*The terms element and factor are used synonymously through the text, except in discussion of factor analysis as a mathematical method of analysis.

of source and supply directly as E_1, E_2, ..., and even to insert some biological factors, B_1, B_2, ... insofar as they may influence the overall phenomenon.

SOME IMPLICATIONS OF THE GENERAL BEACH MODEL

Geologists and engineers have recognized for many years that natural beach processes are complex, and early attempts to resolve problems associated with beach phenomena followed two general paths, which may be referred to as the qualitative geological approach and the quantitative engineering approach. A contrast of these methods is instructive as a background to present day analysis of the same phenomena.

The geologist, operating in an essentially qualitative framework of analysis, and recognizing the multivariate nature of his problem, saw the need for "sorting out" from the complex of interacting factors those features that seemed most strongly to control the general beach process. Observational data were weighed by comparison and contrast among groups of related phenomena. The internal consistency of the data in each group, and the implications of several lines of evidence simultaneously considered, commonly provided a basis for selecting some single set of conditions that most satisfactorily accounted for the phenomena of interest. Chamberlin years ago (1897) described this method of multiple working hypotheses in what has become a classical paper on geologic methods of reasoning.

In his qualitative framework the geologist recognized that the characteristics of a given beach depend strongly on the geological setting of the beach area. Beach material was seen as being fed to the beach by streams or by wave erosion of banks and cliffs. Moreover, it was clearly recognized that period and height of the waves and their angle of approach are important controlling factors in the characteristics of the beach and of the changing features of beach material. The effects of wave refraction in these processes were recognized, at least qualitatively. Davis, for example, gives an illustration of a pattern of wave refraction, as reproduced by Johnson (1919, p. 75).

An important feature in the geological analysis of beach phenomena was recognition that beaches go through several stages of development with time. Johnson, in his classic book on shore processes (1919) elaborates this theme in detail, and describes stages in the shore cycle as exhibited by coastlines of submergence and emergence.

The qualitative geological framework of analysis, though it provided satisfactory hypotheses for the explanation of beach phenomena through time, lacked almost completely the kind of numerical analysis that is required for predicting over short periods the stability of beaches under different wave conditions. Thus, although a wealth of carefully analyzed and integrated qualitative information was made available on beaches, this body of knowledge provided mainly a framework of reference in the more quantitative approach to beach phenomena that was needed for design of coastal engineering structures.

Because of their need to design structures that can withstand wave action, engineers from the start emphasized numerical measurement of coastal phenomena. Because of the many factors operating simultaneously on natural coasts, wave tanks were ultimately designed to permit controlled experimentation on those particular factors which appeared to be of most importance for engineering application. The wave tank experiments permitted a controlled "sorting out" of the variables in the general beach process, and provided a basis for quantitative evaluation and measurement of selected aspects of natural beaches.

In early wave tank experiments sand sieved to uniform grade was emplaced with a fixed initial slope along one edge of the tank, and machine-generated waves approached them head on. By setting the machines to generate waves of fixed height and period, the response of beach slope, say, during a succession of levels of activity of the wave characteristics could be studied. Mathematical functions of the relations thus observed could be developed.

Inasmuch as wave tanks represent scaled down versions of natural beaches, scale model theory is generally required. In order to measure observed effects independently of scale factors, dimensional analysis is used to obtain non-dimensional parameters that can be analyzed directly. For example, some fixed slope s_o may be assigned to the initial beach, and the actual slope s at equilibrium for a given set of wave characteristics, may then be expressed as the ratio s/s_o. In similar fashion, the process factors wave height and period can be converted algebraically into the non-dimensional ratio of wave height to length, H/L, a basic parameter in wave studies.

It may be seen that in terms of the implicit function of equation (1) this kind of analysis represents the extraction of an explicit model of the following form:

$$G_1 = f(P_1, P_2) \qquad \ldots (2)$$

In equation (2) one of the geometrical attributes of the beach deposit, in this case beach slope, is studied under the influence of two wave characteristics, period and height. In terms of the actual experiment these relations would be shown in terms of s/s_o, and H/L. In its non-dimensional form, the expression would be univariate and of the form $s/s_o = f(H/L)$.

Other examples of wave tank experiments include measurement of sand movement along the wave-tank beach. For this purpose the initial beach is set at some fixed angle to the wave machines, and the amount of sand transported from the up-beach end during each fixed level of wave height and period is measured at intervals by appropriate methods. This also represents a univariate model, in that the quantity of sand moved, Q, (or some non-dimensional version of it) is studied as a function of H/L.

A time factor is implied in expressions of the kind described by equation (2). This may be made explicit by re-expressing the equation as:

$$G_1 = f(P_1, P_2, T) \qquad \text{. . . (2a)}$$

thus bringing the factor of elapsed time for attaining equilibrium into the analysis.

By using sand with a range of particle sizes in wave tank studies, it is possible simultaneously to investigate beach slope as well as the selective wave distribution of particular grain sizes across the beach, as a function of wave height and period. This particular experiment would be a multivariate model of the following form in terms of equation (1):

$$(p_1, G_1) = f(P_1, P_2) \qquad \text{. . . (3)}$$

Equation (3) is interesting because it contains both a particle property and a geometrical property of the beach as a function of two process factors. In its non-dimensional form such an experiment would be reduced to $(s/s_o, D_m/D_o) = f(H/L)$, where D_m is the mean particle size on any part of the beach in relation to the initial mean particle size D_o of the well-mixed sand. As with equation (2), this kind of experiment may explicitly include the time factor as illustrated in equation (2a).

Although the relations shown in equations (2) and (3) may be made the basis for experiments to obtain empirical knowledge about beach processes, they may also be set up on the basis of some underlying theory that predicts what these relations should be. In the latter case the experiment tests the theoretical statement. Thus, beach models for wave tank studies may be empirical or analytical depending on the background of theory available for predicting certain physical or mathematical relations between process factors and beach responses.

Observational data from natural beaches fit easily into this same framework of empirical or analytical relationships, giving rise to the derivation of statistical or conceptual physical models from equation (1). For example, studies of the relation between foreshore slope and average grain size of the foreshore (see Figure 1) represent an empirical model derivable from equation (1), in which a geometrical property of the beach is studied as a function of a particle property:

$$G_1 = f(p_1) \qquad \text{. . . (4)}$$

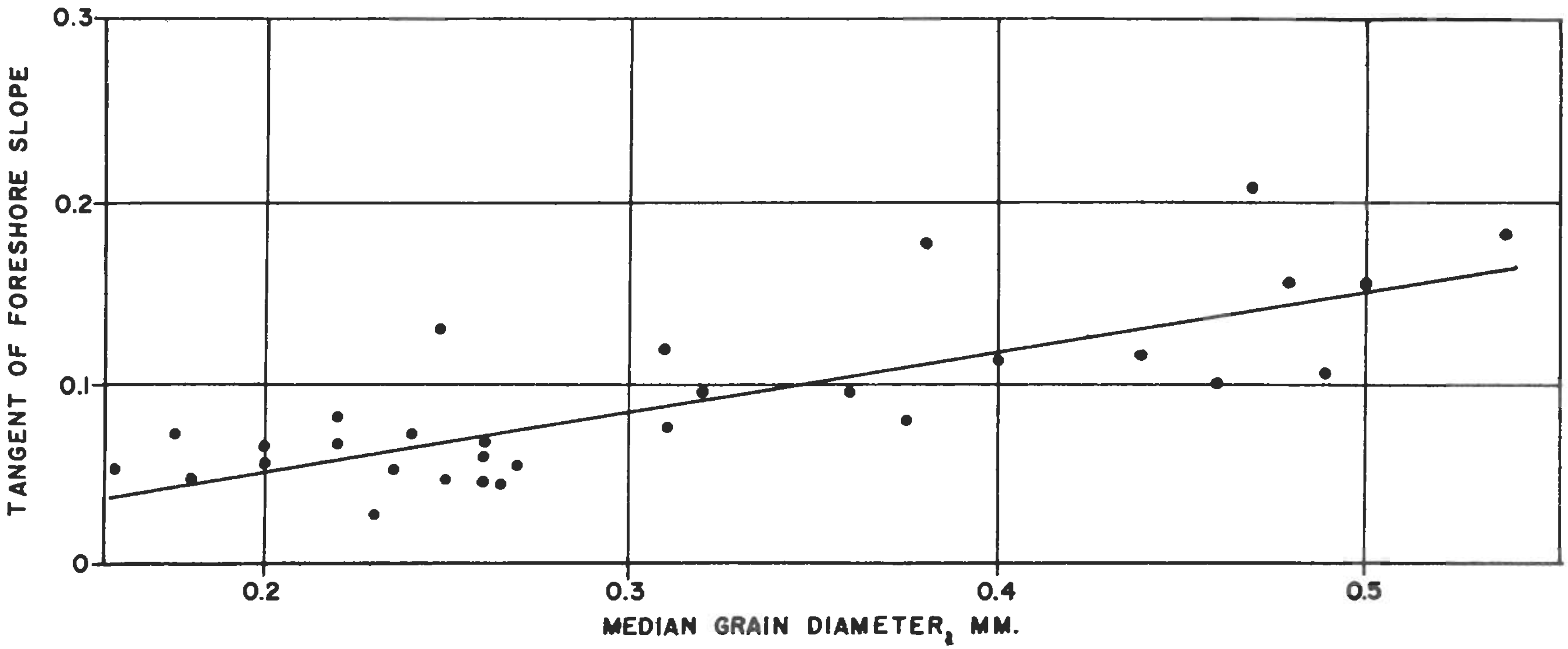

FIGURE I

Relation of Foreshore Slope to Median Diameter of Sand Grains, from Beach Erosion Board Interim Report, 1933, Fig. VII.

Even though observations were made on a number of natural foreshores under varying wave conditions, the linear relation shown in Figure 1 was obtained despite variations in space, time, and wave conditions, which may be thought of as "noise" presumably introduced by these extraneous factors. The explicit relationship for the observations may be stated as:

$$s = f(D_m) = a + b(D_m) + e \qquad \ldots (4a)$$

where s is foreshore slope, D_m is the mean grain size, a is the Y-intercept at X = 0 in Figure 1, and b is the slope of the straight line. The e represents the scatter of the points about the straight line, and may be taken to represent a random component in the observations, as will be discussed later.

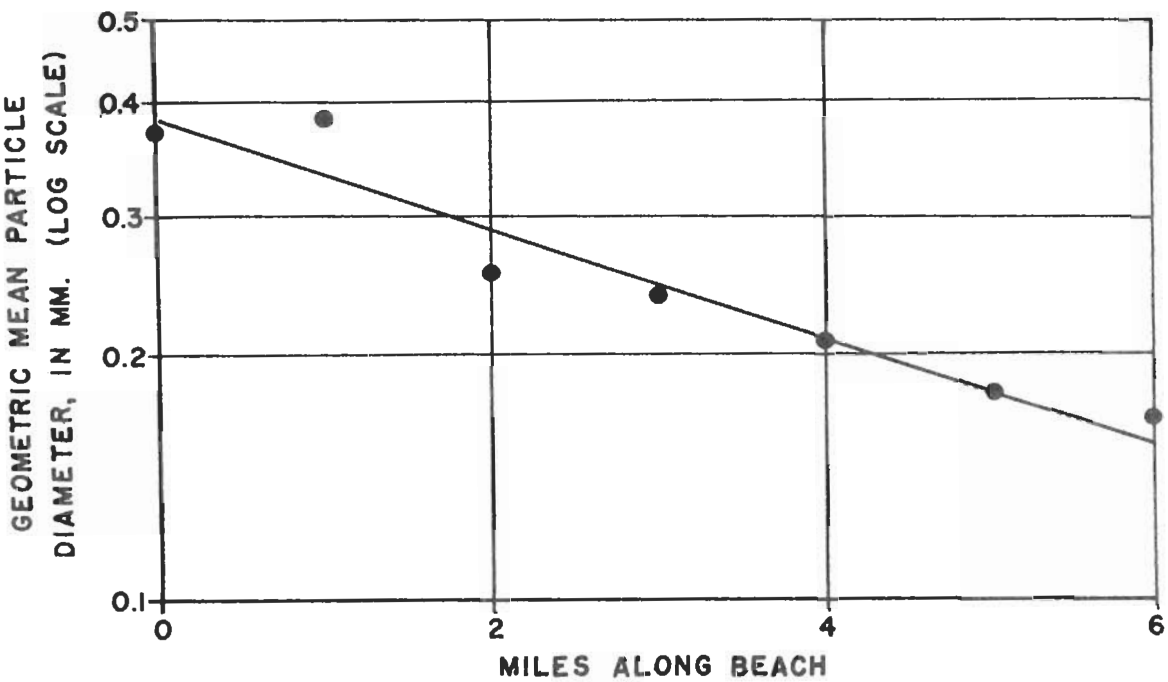

FIGURE 2

Geometric Mean Diameter of Beach Sand as a function of Distance along Beach, from Pettijohn and Lundahl, 1943.

Starting approximately in the 1930's, geologists began studying beach phenomena quantitatively. A common type of study involved examination of the changes in average particle size along the extent of a beach. Figure 2 is an illustration based on Pettijohn and Lundahl (1943). Such experiments tended to show that the decrease in grain size is exponential with distance. The study represents another univariate model extracted from equation (1) in which a particle property is studied in relation to one geographic coordinate, S_1. The corresponding function would therefore be of the form:

$$D_m = f(S_1) \qquad \ldots (5)$$

For studies illustrated by equation (5), recognition that the relationship tends to be exponential permits more explicit expression of the mathematical function as:

$$D_m = D_o e^{-aS_1} \qquad \ldots (5a)$$

where D_m is the mean grain size at any given distance S_1 along the beach, D_o is the initial mean size at $S_1 = 0$, e is the constant 2.71828, and the coefficient a indicates the rate at which average particle size decreases. From such relations it is also possible to obtain an underlying differential equation, of the form:

$$dD_m/dS_1 = -a(D_m) \qquad \ldots (5b)$$

which indicates that the rate of size decrease is a function of the mean particle size at any given point along the beach.

A natural extension of beach studies illustrated by equation (5) is use of a beach grid designed either as a rectangular sample layout or as a series of beach profiles spaced along the beach. Such arrangements permit measurement of the across-beach changes as well as the along-beach changes. In such studies equation (5) includes two geographic coordinates and has the form $D_m = f(S_1, S_2)$.

The advantage of beach grids is that they include an area on which the measured properties can be shown as contour-type maps. For detailed studies the grids are usually set up with square cells to cover a relatively short stretch of beach. However, inasmuch as changes across the beach are commonly much greater than changes along the beach for any unit distance (Krumbein and Miller, 1953), a satisfactory manner of arranging the grid is to set up a series of profiles or traverses at intervals of 1,000 feet or more along the beach, extending from some position on the backshore to some moderate water depth seaward of the low tide line.

In order to maintain comparable positions in the grid, each of the traverses normal to the shoreline can be subdivided according to the position of the berm, mean sea level, low tide line, and nearshore water depths either at fixed distances from the low tide line, or in equal increments of water depth. Such an arrangement is illustrated in Figure 3.

The advantages of maintaining the beach grid in rectangular form is that it provides a set of samples comparable in terms of their position with respect to the reference lines of the berm and the low water line. Such a rectangular grid also facilitates formal mathematical or statistical analysis of the map data.

The samples collected at each grid point may be analyzed for a number of properties. These represent variables in the first part of Table 1, and may include in a single geological study the mean particle diameter, the degree of sand sorting, the mean particle shape, mineralogical composition, bulk density, and so on, measured for each sample on the grid. Various geometrical properties of the beach may also be mapped, such as foreshore slope, beach firmness, or other properties measured at a number of grid sampling points. Figure 4 is an example of a beach firmness map.

Commonly each of the measured attributes is mapped separately as a contour-type map. Thus, although each map represents a single variable, in the aggregate a set of maps permits "mental integration" of the data into a multivariate whole.

Inasmuch as maps are among the most powerful means for communicating geological information, it would be desirable to develop a general multivariate method of mapping, in which a number of attributes could be shown with a single set of contour lines. This is possible for data expressed in percentages where N variables that add to 100 percent can be mapped as a single contour system by transforming the data in special ways (Pelto, 1954).

Among mapping problems that require attention and which can be applied to univariate or multivariate maps, is that of map comparison. If two maps look much alike, one may ask whether this similarity is due to the fact that both mapped variables are responding in a similar manner to the causative factors, or whether, because of a high statistical correlation between the two variables themselves, one of them merely repeats information supplied by the other. This is the general problem of data redundancy, which applies to map data as well as to other kinds of measurements.

To a large degree maps of beach attributes, either of particle properties or geometrical characteristics of the beach itself, are empirical

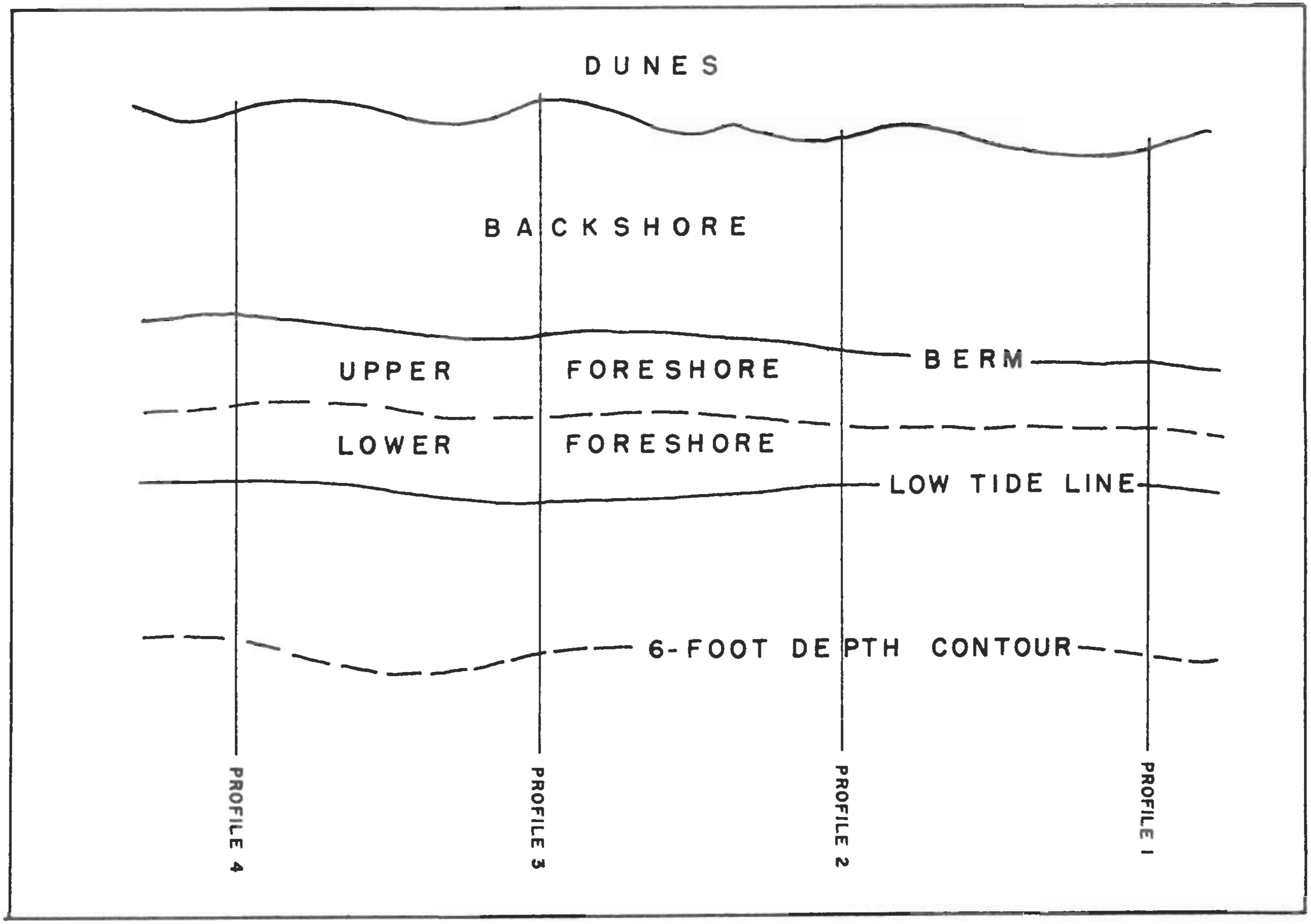

FIGURE 3

Schematic Layout of Traverses on Beach in Relation to Several Beach Zones

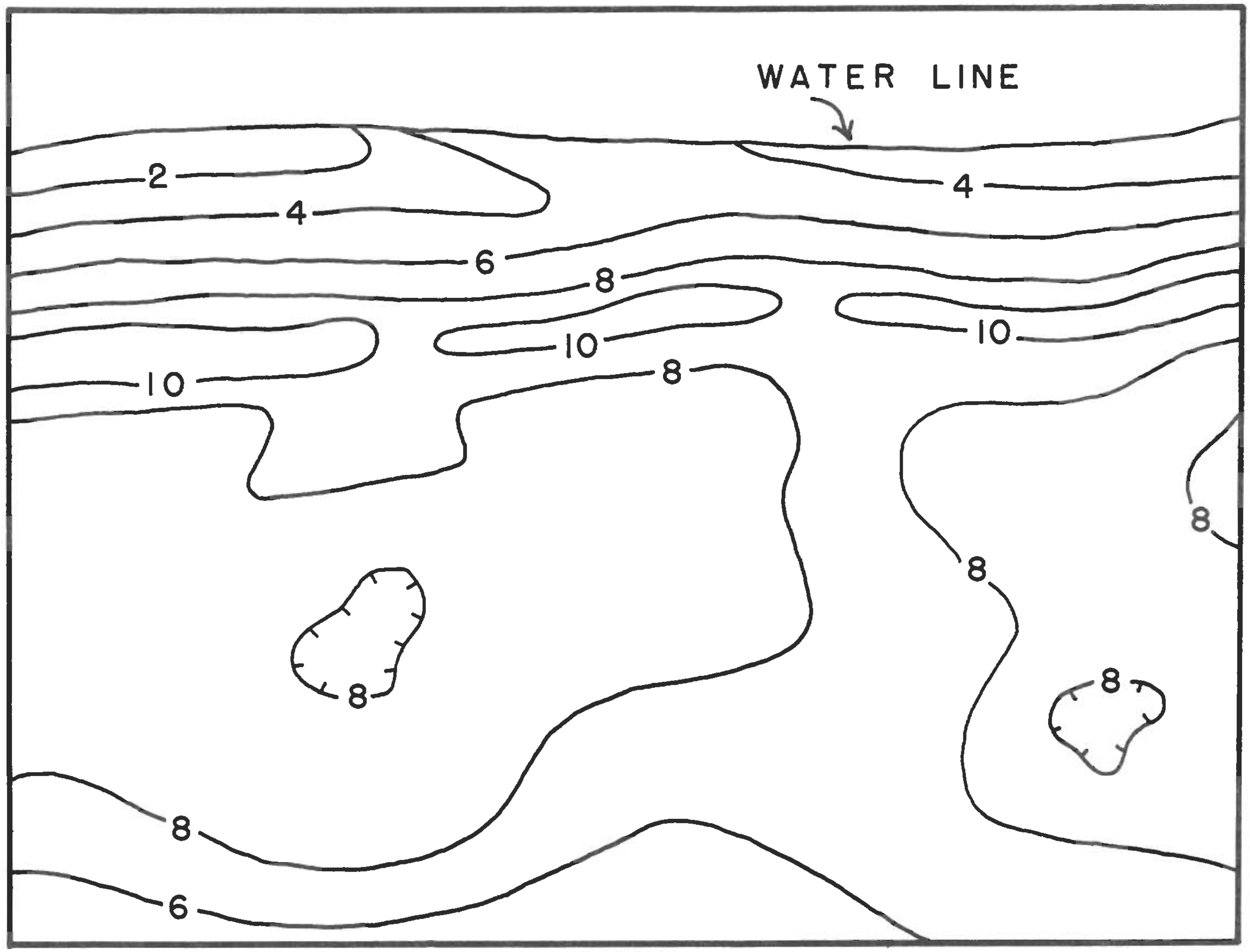

FIGURE 4

Generalized Beach Penetrability Map, Lake Michigan Beach near Evanston, Illinois. The softest areas (contour value 10) occur along the dry berm crest, and the firmest areas occur in moist sand near the water line. Length of beach segment about 100 feet.

devices for examining the state of the beach at given times, to learn how beach attributes vary areally under the influence of natural forces. Maps may also be thought of, however, as devices for testing predictions that arise from conceptual physical beach models.

An example of such a model is furnished by Miller and Zeigler (1958). These authors selected the region of shoaling waves, breaker zone, and foreshore as a special part of the general beach environment that represents an important zone in energy transformation. Observational data included measurements of wave height, length, and period, still-water depth and level, as well as average velocity of backwash on the foreshore. These in general represent beach process factors. Responses that were measured include the foreshore slope; and the mean grain size and degree of sorting of the beach sand.

The conceptual model included assumptions as to the grain size pattern left just after a given uprush passed over the foreshore; consideration of the velocity required to start a grain of given size moving downslope with the backwash; the velocity distribution of uprush and backwash on the foreshore; and computation of the theoretical contours of particle size and sorting as they are distributed over the beach zone of interest.

As a result of their model study, Miller and Zeigler present postulated maps of particle properties for the nearshore breaker zone and foreshore as they occur in equilibrium with wave and current dynamics. The maps are shown as block diagrams in Figure 5.

Miller and Zeigler's conceptual model is thus a predicting device in that it seeks to determine a map pattern on the basis of beach dynamics, such that this predicted pattern can be tested by observational data. In terms of the general relation in equation (1), Miller and Zeigler's model is essentially of the following form:

$$(p_1, p_2, G_1) = f(P_1, \ldots, P_n, S_1, S_2) \qquad \ldots (6)$$

in which two particle properties and one geometrical beach property represent the areal variations of selected beach responses to some half dozen beach process factors. The dependent variables representing particle properties are then mapped as univariate features responding to a multivariate process complex.

In summary of this section it may be re-emphasized that although the general beach process represents very complicated relationships among a large number of interlocking variables, various parts of the process can be selected for analysis on any of several bases, including wave tank experimentation and field studies on natural beaches. In this sense equation (1) is mainly a framework for structuring such studies in terms

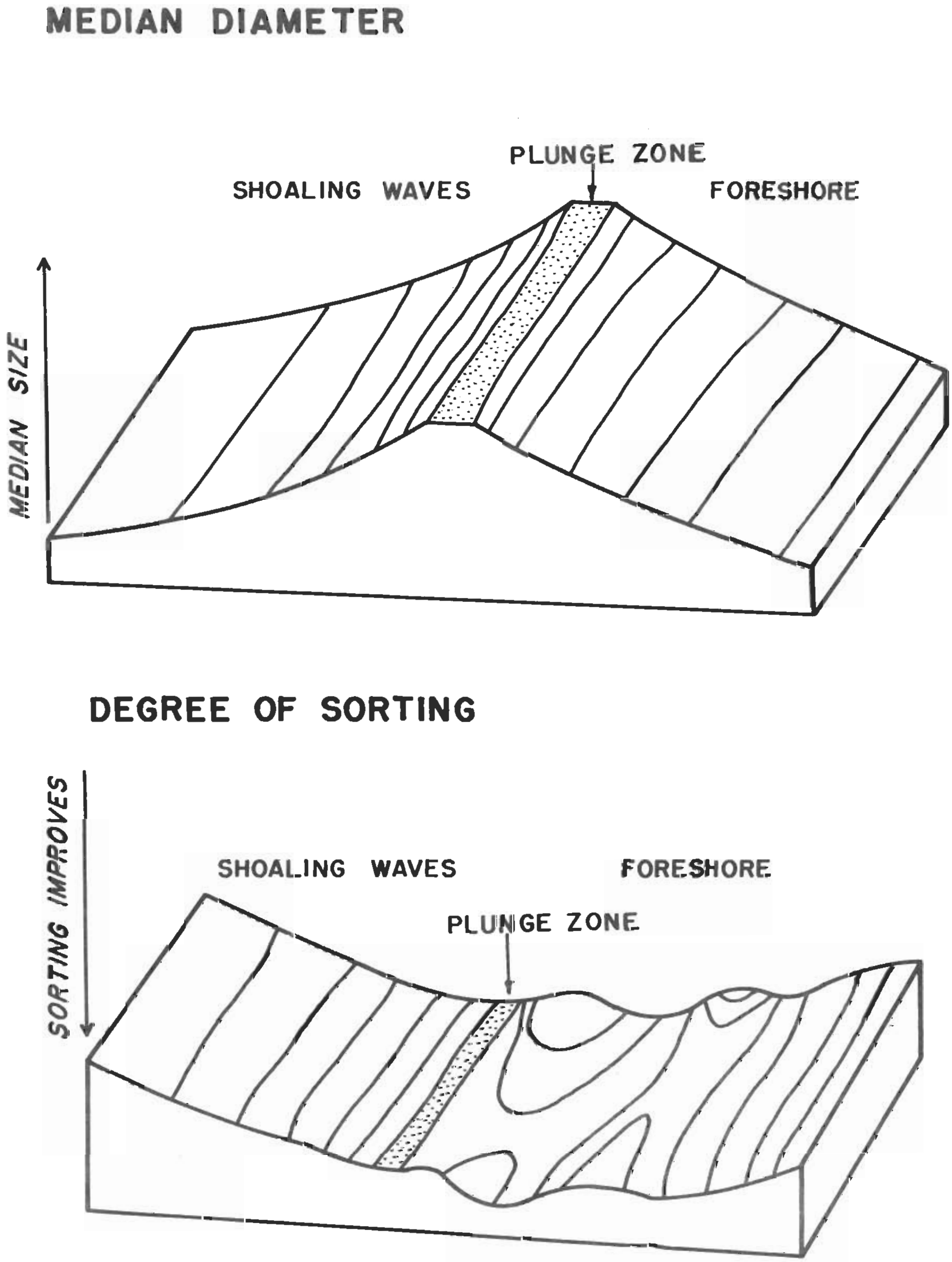

FIGURE 5

Trend Surfaces of Median Diameter and Sorting of Beach Sand in Zone between Lower Foreshore and Nearshore Bottom. From From Miller and Zeigler, 1958.

of responses that arise from the operation of beach processes over an area and through time.

Although one may formally define the framework of a given beach study in terms of beach processes and beach responses in a particular beach area during some given interval of time, it is not possible in any design (on the basis of man-hours or dollar-cost) to measure every individual item that bears on the general beach process. Such a complete study would include not only all the main features of the beach complex, but even the small-scaled local fluctuations that occur sporadically in space and time.

Each feature that is not measured in any study contributes some share of "unexplained" variability to the observational data, and everything that is measured contributes some share of measurement error. As a result, the design element in beach studies rises to major importance in order to assure that the unexplained sources of variability are not so great that they obscure the very relations that are being sought, and that measurement error does not contribute conspicuously to the "noise level".

In the remaining sections of this report some problems of extracting meaningful information from observations on natural beaches will be considered. In particular, the vexing problem of "noisy data" will be treated in some detail.

THE PROBLEM OF HIGHLY VARIABLE NATURAL BEACH DATA

It is apparent from the foregoing that observational data on natural beaches is subject to some degree of "noise" in the sense that a residue of "unexplained variability" usually remains after formal analysis has been completed. As stated, this residual is in part related to measurement error, to sampling error, to local fluctuations along the beach in time, and to those elements of the general process that are not included in the study.

In some studies the largest contribution to the "unexplained variability" may be made by factors not explicitly included in the study. If this variability gets "out of hand" it may be difficult to extract meaningful generalizations or predictions from the investigation. Even for those features that are measured, it is important to design the study so that the contribution of each can be "sorted out" during analysis of the data.

It is sometimes advantageous to design a natural beach study around a particular response that may be of major interest. Thus, if the foreshore slope is to be studied as a response to beach process factors, the statistical or conceptual physical model can be so designed that all major factors controlling the slope are included in the analysis. Whatever these controlling factors may be, optimum design requires that they be measured simultaneously. If the selection of the variables is physically sound, this simultaneous measurement provides a basis for evaluating the contribution of each process factor in the presence of the other process factors. Moreover, judicious selection of variables helps keep the level of "unexplained variability" to a minimum.

Even with optimum design there is no necessary assurance that in the present state of knowledge all beach studies will be successful. Much depends upon unpredictable events that may occur in the beach area, such as the weather, rapid changes in wave characteristics or angles of approach, and so on. Such events may keep the beach in such a state of flux during the study that equilibrium conditions are not attained in terms of any given set of events. One result of such situations may be that the "noise level" of the data may become very high.

We may digress for a moment to consider a wave tank experiment on foreshore slope. Here, as stated earlier, the initial state of the experiment is controlled, inasmuch as the beach is given a fixed initial slope, waves approach at a fixed angle, and the characteristics of the waves remain constant for any one experimental run. It is observable during the course of such experiments that the slope is modified by the waves and that it gradually assumes an equilibrium state adjusted to the particular conditions of the experiment. A second experimental run, with different wave characteristics, but with the same initial slope as in the first run, will also attain equilibrium under the new experimental conditions. In this way one can study the changes in beach slope resulting from particular wave conditions persisting for given lengths of time.

Inasmuch as the "initial state" in a field experiment is simply that foreshore slope which natural forces had developed by the time the observations start, and because this inherited condition may persist for some time even after the wave conditions change, there may be no assurance that the foreshore slope observed at a given moment may be in equilibrium with the process factors operative at that moment. Moreover, if wave conditions change fairly rapidly during a field study, the foreshore slope may fluctuate around several positions of quasi-stability without quite settling down to any one equilibrium state. This sort of situation could also give rise to a high "noise level".

It would appear from these considerations that beach slope studies require a sufficient span of time so that the "inheritance factor" can be evaluated. Thus, one may make observations on process elements for several days preceding the actual slope measurement, as a basis for determining the extent to which earlier wave states control the foreshore slope at a given instant.

In considering the problem of designing studies for natural beaches, it seemed important, when this report was first contemplated in 1957, that this question of a time lag in beach responses should be examined at least in a preliminary way. Inasmuch as the foreshore slope is an important engineering consideration, it was thought that sets of data would be available in the files of the Beach Erosion Board that could be used for a "trial run" on the time-response of natural foreshore slope to beach process factors.

As a guide for data searching, it was considered desirable to have a set of data for a given beach over several seasons of the year, in which beach foreshore slope at some given point was measured, and in which data were available on beach process factors for at least a few days previous to the time when the beach response was observed. Somewhat surprisingly, it was found that data of this sort, which involve a series of process factors measured simultaneously over a period of time preceding measurement of beach properties, is rare among beach records. One set of data came fairly close to the requirements. These were measurements made at Mission Beach, California, during 1950 and 1951, in a study of methods for measuring water depth in the nearshore zone (Saville and Caldwell, 1953). Out of these records it was possible to assemble subsets of data covering 23 days scattered over the interval February 8, 1950, to April 25, 1951.

From this set of data it was possible to set up a conceptual field model that involved measurement of the foreshore slope at Zero Hour, and that included measurement of several beach process factors at Hours -6, -24, -30, -48, and -54. Thus the foreshore slope observed at Zero Hour could be studied in its relationship to process elements as they occurred over the preceding 54 hours. If the process elements operative shortly before the time of measurement had the strongest effect on beach slope, it may be expected that the influence of these same process elements backward through time should diminish asymptotically toward zero.

In the original study a series of profiles was surveyed across the beach to the 30-foot water depth, from which the average foreshore slope could be measured. Three of these were chosen for the present analysis in order to have some information on the influence of local variations in foreshore slope along the beach. The process elements were measured from fixed points along the beach, and provided data on wave height, wave period, angle of wave approach, velocity and direction of shore current, and state of the tide.

Measurement of the process factors was approximate rather than accurate, in part because the original study did not require close measurement of these factors, and partly because present-day instrumental methods were mainly in a developmental stage a decade ago. This is not to say, by any means, that the original data are not of high quality for purposes of the original depth study; the point of emphasis here is that each beach study requires its own central design, and the design for one purpose may not be optimum for another purpose.

ANALYSIS OF THE MISSION BEACH DATA

Introduction. As the present study assumed form, it was contemplated that several objectives might be satisfied with a single set of observations. These were (1) the question of the time-lag in foreshore slope response; (2) some evaluation of the relative importance of the beach process elements in controlling foreshore slope; (3) examination of the combined effects of two or more process elements considered simultaneously; and (4) analysis of ways in which the general "noise level" of a set of natural data could be reduced.

Figure 6 is a map of the study area, showing the three ranges involved in the analysis, along which the foreshore slope was measured at Zero Hour. The locations of fixed stations for observations on wave height and period, direction of wave approach, and of shore currents, are shown on the map. The state of the sea was estimated visually; wave height is believed to be accurate to ± 1 foot, and wave period to ± 1.5 seconds. The direction of wave travel is expressed in degrees clockwise from the south. These directions are believed to be accurate to ± 5 degrees. The littoral currents were measured with fluorescein dye, with a relative error of perhaps 15%. The foreshore slope was taken as the average slope from MHHW to MLLW; these were measured from surveyed traverses and are believed to be accurate to ± 0.5 degree. Slope is expressed as the cotangent of the slope angle. Tidal data were not included in this analysis, but will be mentioned later.

TABLE 2

RANGE OF VALUES OBSERVED IN MISSION BEACH
(Complete Data in Appendix, Tables A - G)

Item	Minimum Value	Maximum Value
Wave Height	0.3 Foot	3.0 Feet
Wave Period	6.0 Sec.	20.0 Sec.
Angle of Wave Approach	180 Degrees*	290 Degrees*
Shore Current Velocity	76 ft/min North	123 ft/min South
Cotangent of Foreshore Slope	28.2	50.1

*As measured, 270 degrees represents wave from due west.

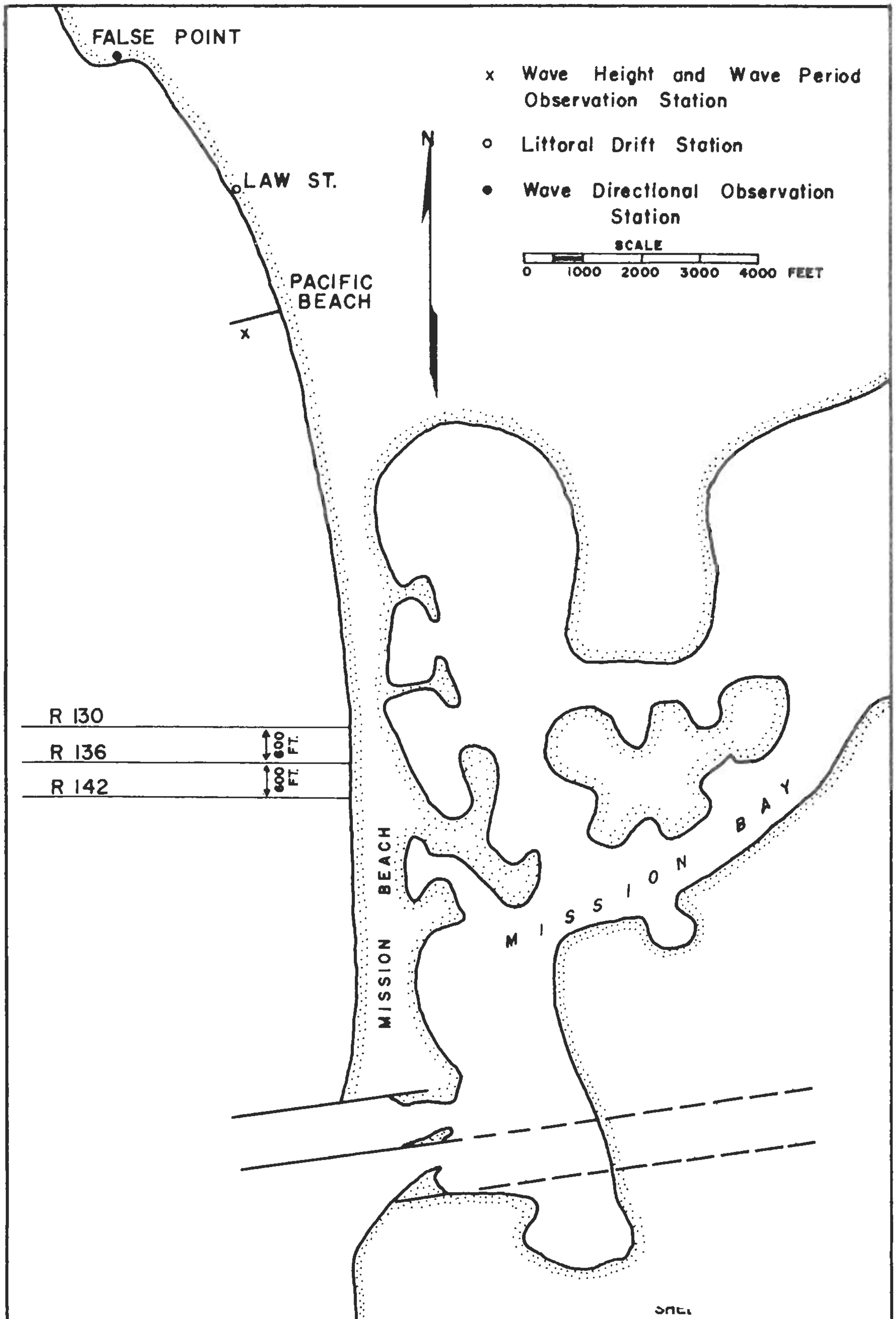

FIGURE 6

Sketch Map of Mission Beach Area, California

Table 2 gives summary information on the range of values observed for the foreshore slope and the several beach process elements. The relatively low maximum wave height is probably a result of the distribution of observation days over the seasons: 3 in winter, 9 in spring, and 11 in summer. The shore current was coded as positive when it flowed northward, and negative when it flowed southward. The complete set of data as used in this analysis is included in Tables A to G in the Appendix to this report.

In terms of equation (1), the conceptual model for the Mission Beach analysis may be expressed in the following form:

$$G_1 = f(P_1, \ldots, P_4, S_1, T_1, \ldots, T_5) \qquad \ldots (7)$$

in that the complete set of data includes one geometric beach property (foreshore slope), considered as a response to four process elements, one geographic coordinate (distance along the beach between the slope measurements), and five points in time.

Preliminary Analysis. In order to set a framework for detailed analysis, several preliminary computations were made on the dependent variable to determine whether any significant differences occurred among the slope measurements at the three beach profiles. It was found that the "within variability" of the 23 measurements at each beach traverse was greater than the "between variability" of the three traverses, which suggested that the differences in slope observed at the three traverses were produced by small-scale fluctuations along the beach rather than by a significant change in slope along the beach.

As a result of this finding, it was decided to average the slope measurements at the three traverses for each of the 23 days. This in effect eliminates the local fluctuations, and stabilizes the slope measurements. Moreover, by averaging the slope at three points along the beach, the S_1 in equation (7) is "eliminated".

Sequential Multiregression Analysis. Several methods are available for analyzing observational data that involve interrelations among the independent variables. Because of the relatively high "noise level" of the present data, even after averaging the slopes at the three traverses, it was decided to explore the possibilities of multiple linear regression analysis. This may be performed by least squares methods without involving any assumptions about the data, although for rigorous statistical interpretation some assumptions need to be considered.

Sequential multiregression analysis involves the study of a given dependent variable (here foreshore slope) in terms of several controlling process elements, by taking the latter one at a time, two at a time, and so on, until all the process elements are included simultaneously.

Regression analysis is most effective when the independent variables can be measured without error, so that their interrelations may be clearly seen, and their relative effects in controlling the response of the dependent variable can be accurately assessed. In the present set of data the measurements of the independent variables (the process elements) are subject to some uncertainty, and the results of the analysis are thereby somewhat clouded. However, it is interesting to observe that certain underlying relations do emerge from the study, and that these relations agree with currently accepted theories of beach processes.

In sequential multiregression the linear relations between the dependent and independent variables are first examined for each independent variable separately. This is accomplished by fitting a least-squares straight line to a scatter diagram of beach slope against each process element in turn. The straight line has the general form

$$Y' = a + bX \qquad \ldots (8)$$

where Y' is the computed value of the straight line at each point of observation of Y; X is the independent variable; a is the intercept of the line on the Y -axis; and b is the slope of the straight line. Thus, the linear regression of foreshore slope on wave height would have the form $s' = a + bH$, where s' is the computed value corresponding to each observed slope value s.

The degree of relationship between two variables can be evaluated by examining the reduction in the sum of squares of the dependent variable produced by the regression function. The total sum of squares for any variable is defined as the sum of the squares of the difference between individual measurements and the mean of all measurements:

$$SS_Y = \sum (Y - \bar{Y})^2 \qquad \ldots (9)$$

where Y is a single observation and $\bar{Y}$ is the arithmetic mean of the observations. The sum of squares of a dependent variable is a measure of its "total variability". This sum of squares can be separated into two portions, one associated with linear regression and the other with deviations of the data from the straight line. Thus, the total sum of squares can be expressed as the sum $SS_Y = SS_L + SS_D$, where SS_L represents that portion of the total variability "accounted for" by the linear relation, and SS_D represents the remaining variability after the linear relation is removed. The percentage reduction in the total sum of squares due to linear regression is then computed from the relation

$$\%\ \text{Reduction in } SS_X = (100)SS_L/SS_Y \qquad \ldots (10)$$

The reduction of the sum of squares is a measure of the mathematical association between variables, and is not necessarily the measure of a physical relationship. However, where the independent variable has physical meaningfulness in the problem, it is not extreme to infer that the strength of the mathematical relation is also a measure of the strength of the physical relation. When the independent variables are taken one at a time, however, interrelations among the independent variables themselves may complicate interpretation of the sum of squares reduction. If a particular independent variable is itself dependent on some other variable, the apparent sum of squares reduction may in large part be influenced by such "hidden" relations. That is to say, the particular independent variable in part repeats information associated with a more meaningful variable, and to that extent it is "redundant".

Because of limitations on regression analysis with one independent variable at a time, sequential multiregression is used to estimate the simultaneous influence of two or more independent variables on the foreshore slope. Sequential multiregression treats the independent variables one at a time, two at a time, and so on for all possible pairs, triplets, etc. of independent variables measured. In this way it is possible to observe how the independent variables affect foreshore slope simultaneously.

In this analysis equation (8) is extended to include additional independent variables. Thus, the linear relation between foreshore slope, and wave height and period simultaneously is:

$$s' = a + bH + cT \qquad \ldots (11)$$

where T is wave period; a is the Y-intercept as before; and b and c are the linear coefficients. The individual values of a, b, and c change as different variables are introduced, and the percentage reduction in the sum of squares associated with each pair indicates how strongly the pair affects slope response simultaneously.

The high-speed computer programs used in this study obtain the linear coefficients and the sum of squares reduction in foreshore slope for all possible combinations of the independent variables from one to four at a time. Thus, in the present study there are four single variable analyses, six analyses representing pairs, four analyses representing triplets, and one analysis representing all four independent variables, for each of the five times of wave and current observation. The results of the analysis and their implications are considered in the next section. Some details of the method are given in Krumbein (1959a).

The Time Lag in Slope Adjustment. The results of the first stage of regression analysis are shown in Table 3. This table shows the extent to which the wave process elements reduce the sum of squares of foreshore slope, when the elements are taken one at a time, for the five times of observation.

TABLE 3

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN ONE AT A TIME

ORIGINAL VARIABLES*

Project	Hour	H	T	a	V
01 0037	- 6	32.49	0.06	0.00	20.14
01 0038	-24	16.90	1.29	2.77	0.08
01 0039	-30	9.73	14.08	21.88	0.90
01 0040	-48	2.01	0.00	11.24	1.79
01 0041	-54	16.72	3.84	6.72	5.39

* In this and subsequent tables H is wave height in feet; T is wave period in seconds; a is angle of wave approach; and V is velocity of the shore current in feet per minute.

The relatively low reduction in the sum of squares and the erratic variations of most of the variables through time suggests either that the regression relations are inherently weak or that the high noise level in the data tends to obscure whatever significant relationships are present.

Perhaps the most consistent variable is wave height, which reduces the sum of squares of beach slope by 32.49% at Hour -6, and diminishes systematically to Hour -48, but rises again rather sharply at Hour -54. Wave velocity, shown in the last column of Table 3, shows a moderate reduction at Hour -6, with wholly negligible reductions farther back in time, except for the small rise at Hour -54. Two of the variables in Table 3, the wave period, and the angle of wave approach, show somewhat surprising results in that they attain maximum values at Hour -30.

If the results of Table 3 can be taken at face value, they suggest that wave height and velocity of the shore current show their most prominent effects shortly before the profiles are measured, whereas the wave period and the angle of wave approach tend to produce their strongest results after a lag of about 30 hours. This point will be returned to in a later stage of the analysis.

Figure 7 is included here to show how weak the regression relations are. The figure shows in its upper graph the regression of foreshore slope against wave height, and in the lower graph is a scatter diagram of foreshore slope against wave period, both graphs representing Hour -6. The data are given in Table A of the Appendix. The straight line in the upper graph reduces the sum of squares of beach slope by 32.49%, as indicated in Table 3. Similarly, the more or less circular cluster of points in the lower graph represents a reduction in the sum of squares of only 0.06%, which means there is no detectable regression relation between foreshore slope and wave period for Hour -6.

Figure 8 is introduced here to show the graphic relationships in Table 3. Each of the process elements is plotted backward in time in terms of the percentage reduction that it produces in the sum of squares of foreshore slope. The curve for wave height drops consistently to Hour -48, and then shows an abrupt rise as mentioned earlier. Current velocity drops abruptly from a maximum at Hour -6, and shows only a slight upturn at Hour -54. Both the angle of wave approach and the wave period show negligible effects at Hour -6, and rise to a maximum at Hour -30, as pointed out previously.

The most pronounced upturn at Hour -54 is shown by wave height. It is difficult to believe that this upturn represents a real effect, in terms both of the consistent decrease to Hour -48, plus the moderate changes in the other variables at Hour -54. Certainly the graphs in Figure 8 do not appear wholly consistent in that two of the variables show their maximum values shortly before the beach slope was measured, and two others suggest a time lag of the order of 30 hours.

The information content in graphs such as Figure 8 may sometimes be shown more sharply when the original variables are expressed in more meaningful transformations. Thus, the dimensionless parameter H/L (the wave steepness) is an important criterion in beach studies. Similarly, some measure of the wave energy may be more revealing than the original raw observations.

inasmuch as both the height and the period of the waves were measured, a transformed variable proportional to wave steepness may be obtained by dividing the wave height by the square of the wave period. Similarly, a variable related to wave energy may be obtained by multiplying the square of the wave height by the square of the wave period.

Figure 9 shows graphs of the transformed variables taken one at a time. The data are listed in Table 4. The energy curve is striking in that it drops abruptly from nearly 30% reduction virtually to zero at Hour -30, and shows only a slight upturn at Hour -54. The wave steepness curve shows only a secondary maximum at Hour -30, and a slightly larger upturn than is shown by energy at Hour -54. Both of these graphs strongly suggest that

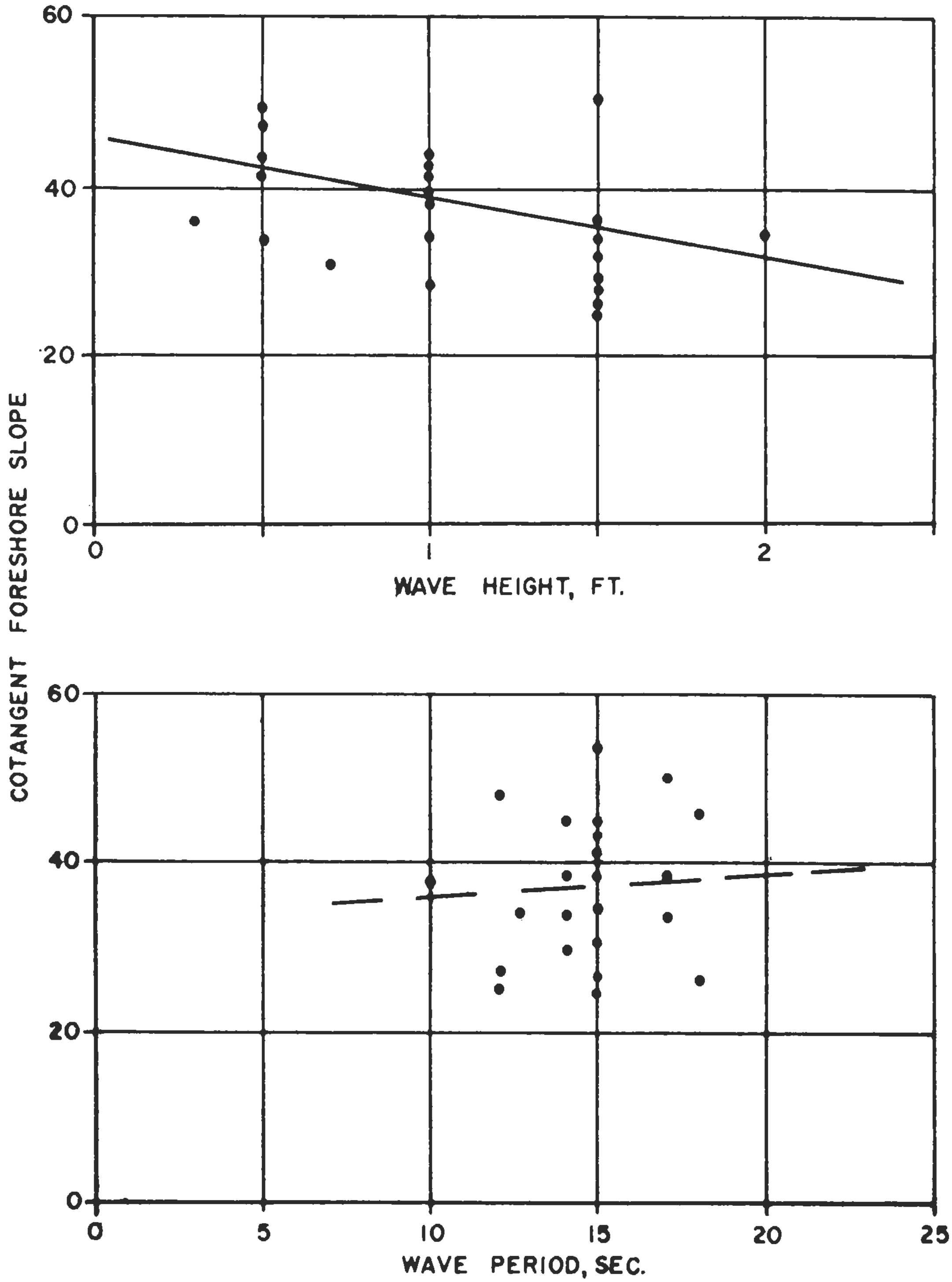

FIGURE 7

Scatter diagrams of Foreshore Slope as a function of Wave Height (upper diagram) and wave period (lower diagram).

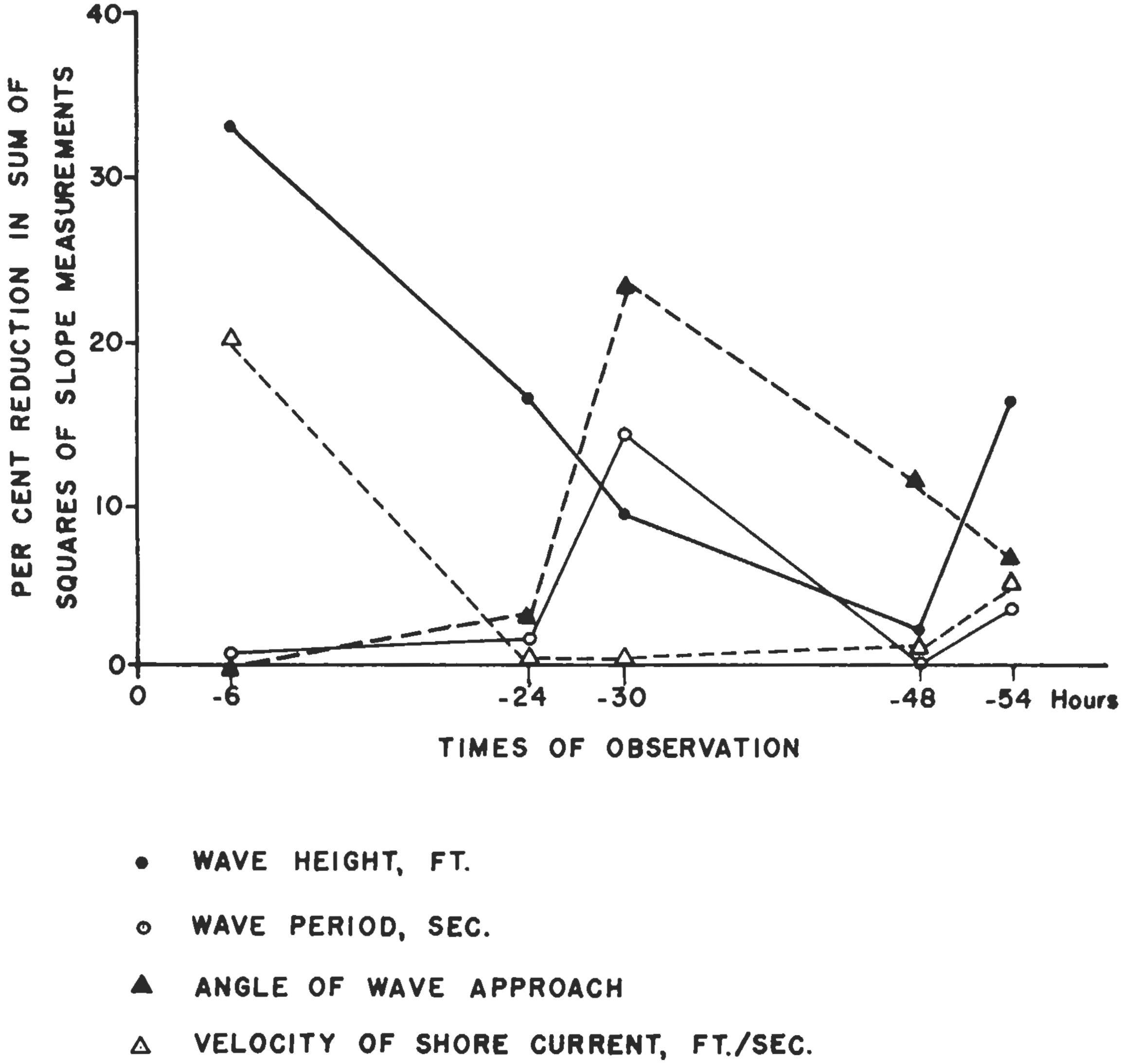

FIGURE 8

Sum-of-squares Reduction through Time for Mission Bay data, Untransformed Variables taken One at a Time. (See Table 3)

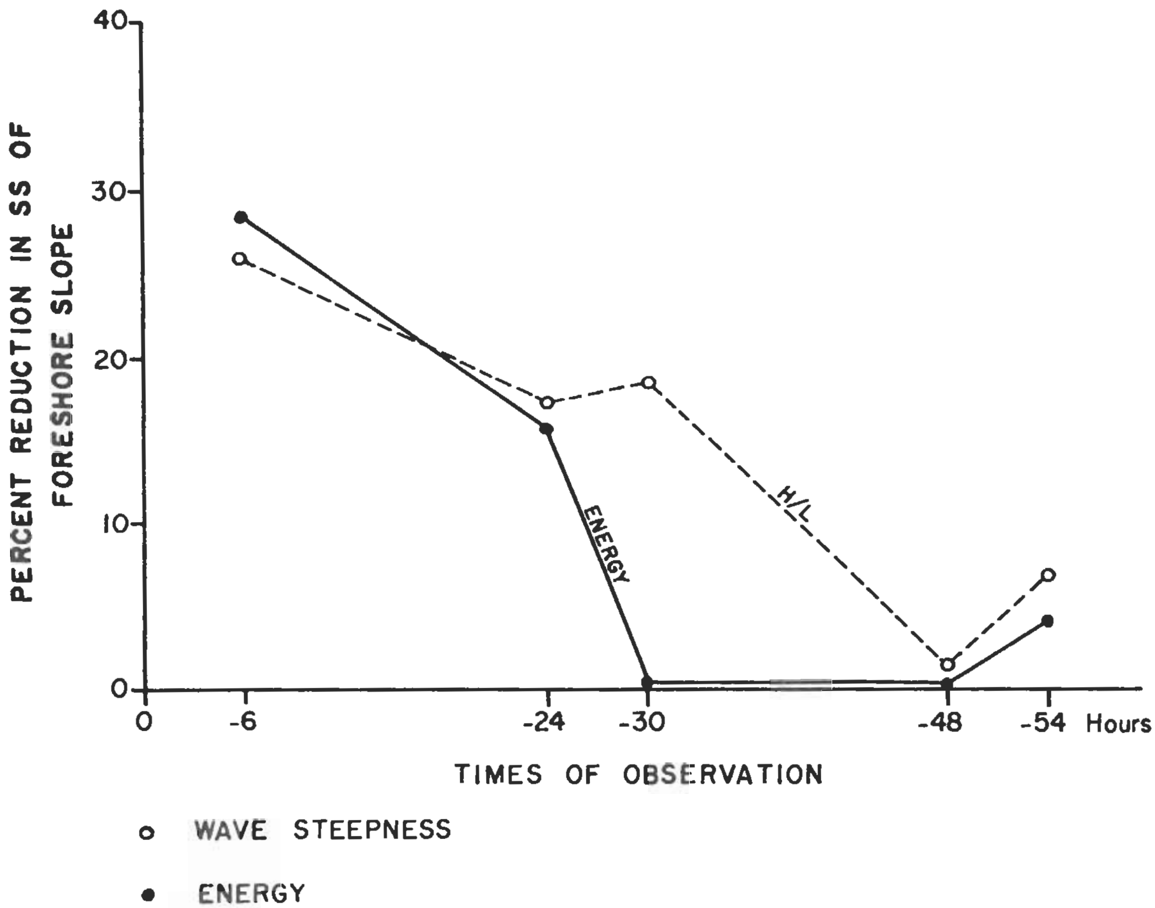

FIGURE 9

Sum-of-squares Reduction through Time for Mission Bay data, Transformed Variables taken One at a Time. (See Table 5) the time lag in slope adjustment occurs within about 48 hours. Considering that the 23 days in this set of data are scattered over more than a year, and include several seasons, it is believed that these graphs yield physically significant information.

TABLE 4

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN ONE AT A TIME

TRANSFORMED VARIABLES*

Project	Hour	H/L	E
01,0037	- 6	25.95	28.69
01 0038	-24	17.49	16.00
01 0039	-30	18.62	0.35
01 0040	-48	1.47	0.06
01 0041	-54	7.26	4.42

* In this and subsequent tables H/L is wave steepness, and E is wave energy.

In the second stage of multiregression analysis, the original process elements are taken in pairs. In one sense the transformed variables of Figure 9 do represent a pairing of the original variables, in that both wave height and period are included. However, this pairing is made on a physical rather than on a statistical basis. It is interesting to consider how the original variables behave when they are directly paired. Four original variables yield six possible pairs, and the sum of squares reduction in foreshore slope associated with the pairs is listed in Table 5. The largest reduction in the sum of squares is associated with the paired wave height and shore current velocity. The next two larger ones are the pairs of wave height and period, and of wave height and angle of approach.

Figure 10 shows graphs of strongest and weakest pairs through time. As may be noted, the curve of wave height and velocity is quite similar to that for wave height alone for Figure 8. The pair with wave period and angle of approach has its maximum value at Hour -30, similar to these variables separately in Figure 8.

When the transformed variables that represent wave steepness and energy are combined, they display the curve in Figure 11. Here the secondary rise at Hour -30 is no longer present, although the sharp rise at Hour -54 is still present. The data supporting Figure 11 is shown in

TABLE 5

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES
ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN TWO AT A TIME

ORIGINAL VARIABLES

Project	Hour	H, T	H, a	H, V	T, a	T, V	a, V
01 0037	- 6	32.52	33.38	36.57	0.07	20.15	20.23
01 0038	-24	17.16	16.97	18.83	2.79	1.75	3.04
01 0039	-30	23.26	26.24	10.57	24.53	15.05	23.10
01 0040	-48	2.01	12.11	4.70	11.86	1.83	22.02
01 0041	-54	17.56	17.35	19.22	6.73	7.79	9.32

TABLE 6

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES
ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN TWO AT A TIME

TRANSFORMED VARIABLES

Project	Hour	E, H/L
01 0037	- 6	34.39
01 0038	-24	21.44
01 0039	-30	20.02
01 0040	-48	1.49
01 0041	-54	11.58

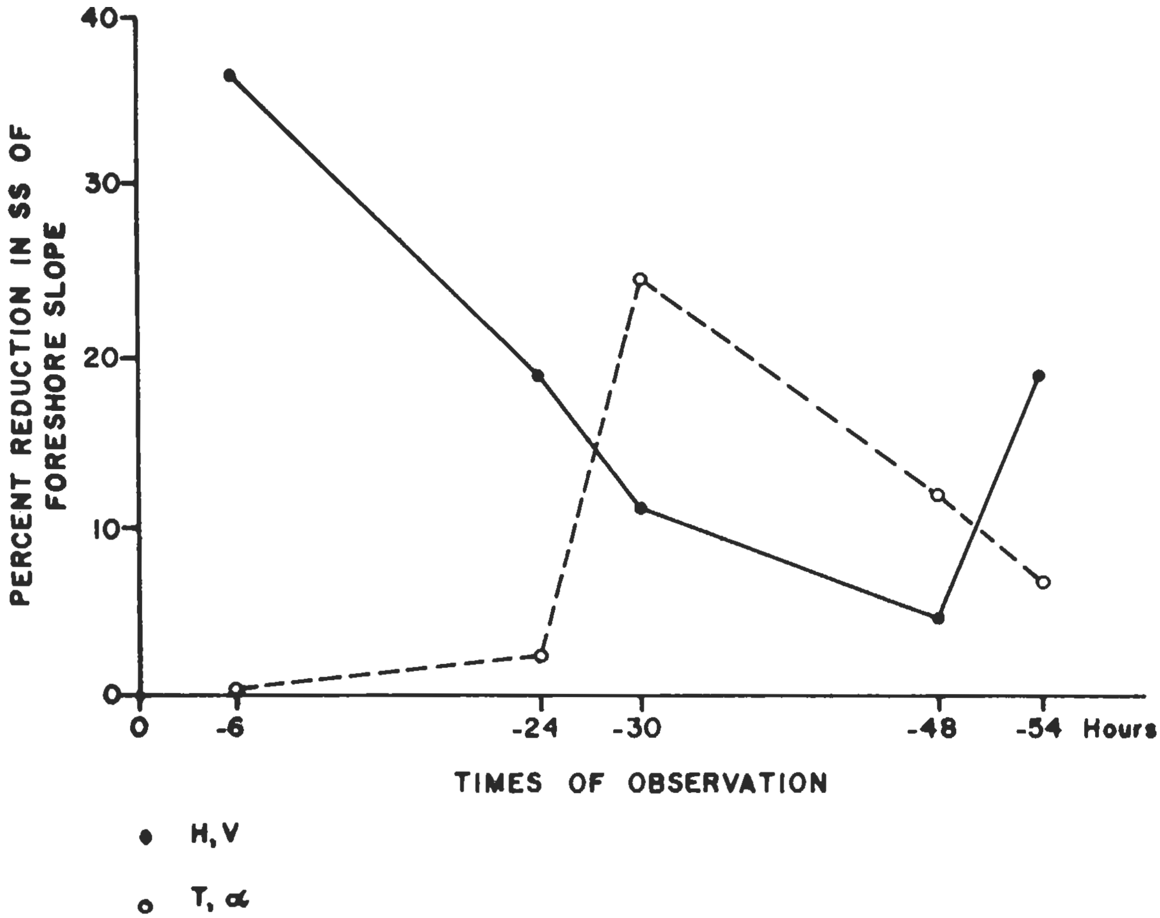

FIGURE 10

Sum-of-squares Reduction through Time for Mission Bay data, Untransformed Variables taken Two at a Time. (See Table 6) Two Examples only.

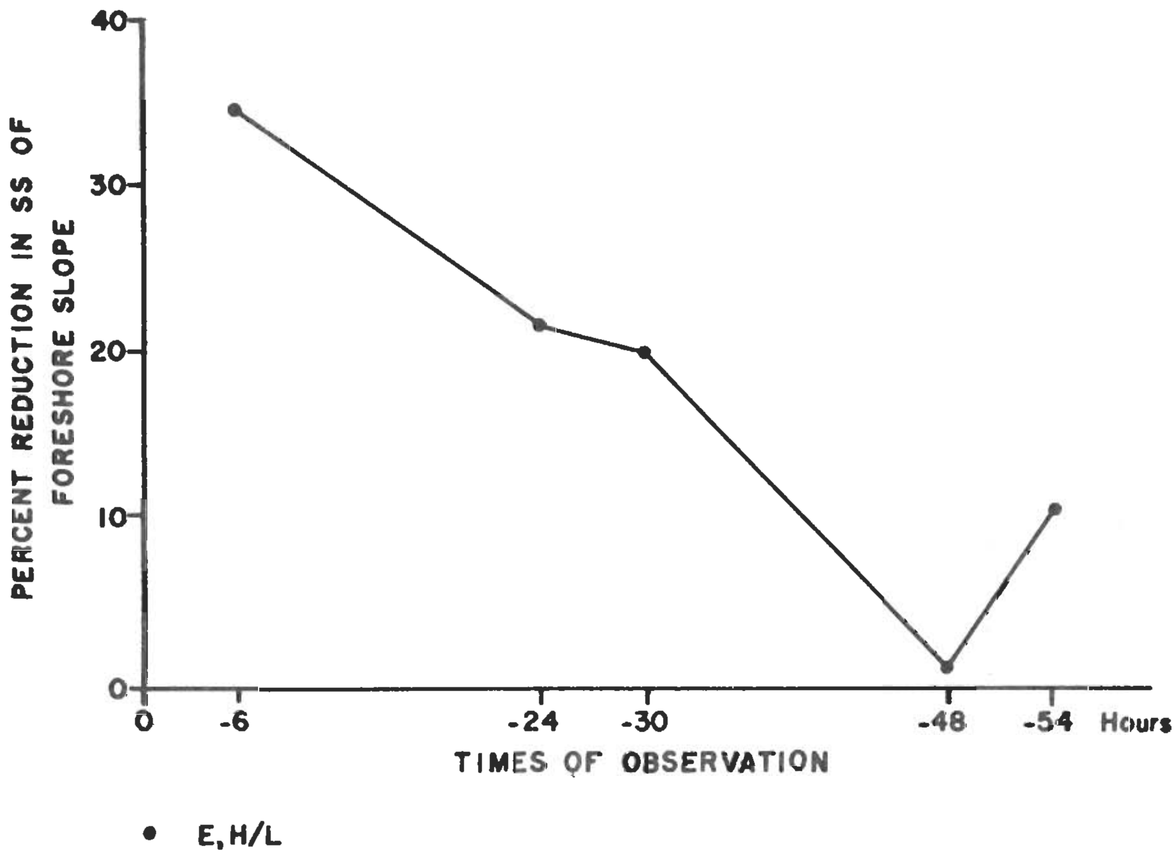

FIGURE 11

Sum-of-squares Reduction through Time for Mission Beach data, Transformed Variables taken Two at a Time. (See Table 7)

Table 6. It is interesting to compare this table with Table 4, to indicate some of the interrelations between the two transformed variables. In Table 4, the reduction in the sum of squares due to wave energy at Hour -6 is 28.69%. That due to the wave steepness in 25.95%. At Hour -6 in Table 6, the reduction due to both of these variables operating simultaneously is 34.39%. If on the basis of Table 4 one accepts wave energy as being the stronger transformed variable, then the contribution of wave steepness in the presence of wave energy is measured by the difference between 34.39% and 28.69%. This amounts to 5.70%, which represents a considerably smaller component than the contribution of 25.95% associated with wave steepness alone.

The changing picture of relative importance in process elements as they are combined in multiregression analysis, is a reflection in part of the amount of information associated with any one variable that is repeated by another. The term "data redundancy" is commonly applied to this phenomenon.

A similar analysis may be conducted on Table 5 in comparison with Table 3. For Hour -6, the reduction in the sum of squares of foreshore slope attributable to wave height and current velocity is 36.57%. If in terms of Table 3 one accepts wave height as the strongest variable, then the contribution of shore current velocity in the presence of wave height is the difference between 36.57% and 32.49%. This difference, amounting to 4.08%, is also a sharp decrease in the reduction of 20.14% which is shown by shore current velocity alone in Table 3.

The exact role of the angle of wave attack in this set of data is somewhat perplexing. One would infer intuitively that this variable should be closely related to the strength of the shore current, and that its effects on foreshore slope should be more nearly coincident with the time of slope measurement, rather than showing a retarded effect of the order of 30 hours. The same argument with respect to apparent retardation applies also to wave period (refer to Figure 8).

When wave height and period are combined in the transformed variables, as shown in Figure 9, the strong maximum at Hour -30 of wave height alone has become only a secondary effect in the H/L curve, and has been lost entirely in the energy curve. Purely as an experiment, the angle of wave approach was paired with the wave steepness, to obtain an equation based on two non-dimensional quantities. The result is shown in Figure 12, where the influence of the angle of approach can be seen in the larger secondary maximum at Hour -30. However, in its total effect Figure 12 suggests a decreasing function backwards through time. The data for Figure 12 are given in Table 7.

Comparison of Tables 4 and 7 show that for Hour -6 the apparent contribution of the angle of wave approach in the presence of H/L increases

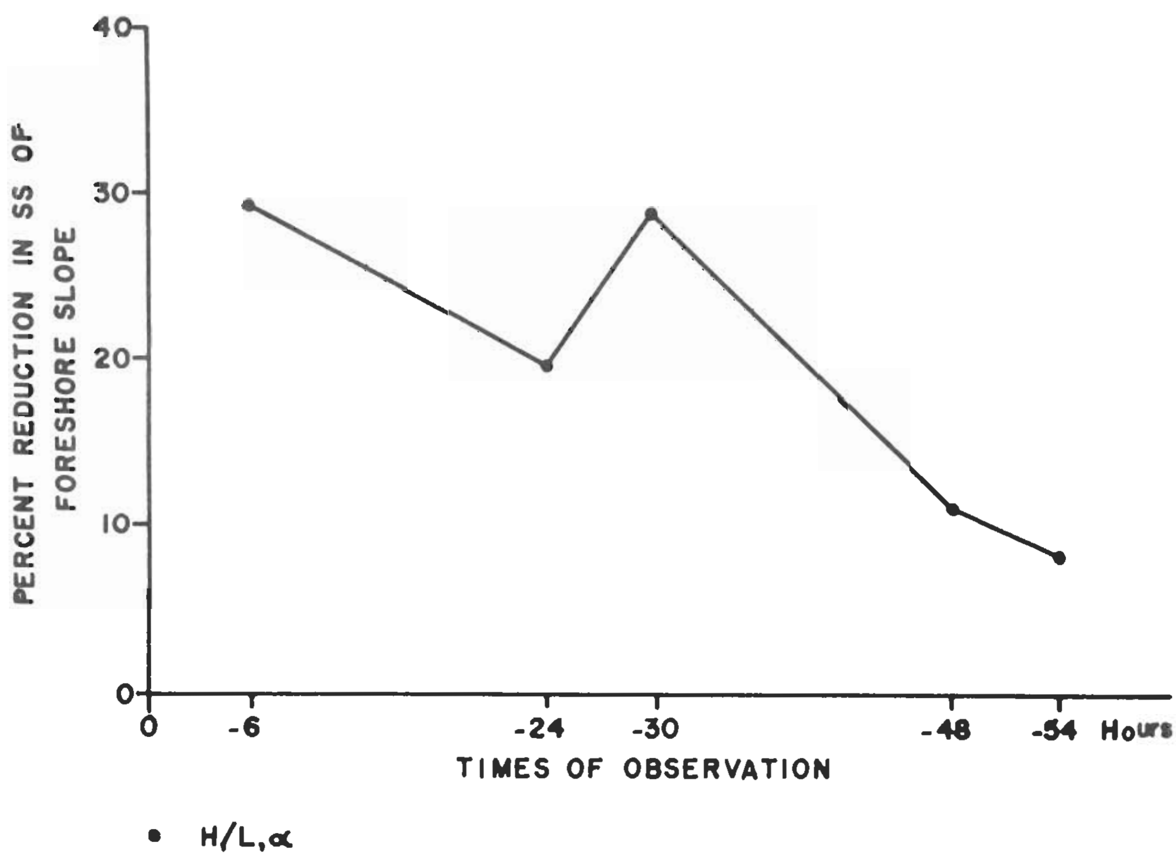

FIGURE 12

Sum-of-squares Reduction through Time for Mission Beach data, Dimensionless Variables (See Table 8)

TABLE 7

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN TWO AT A TIME

DIMENSIONLESS VARIABLES

Project	Hour	H/L, a
01 0037	- 6	29.23
01 0038	-24	19.80
01 0039	-30	29.47
01 0040	-48	11.36
01 0041	-54	8.68

TABLE 8

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN THREE AT A TIME

ORIGINAL AND TRANSFORMED VARIABLES

Project	Hour	H,T,a	H,T,V	H,a,V	T,a,V	H/L,E,a
01 0037	- 6	33.73	36.57	37.43	20.23	36.02
01 0038	-24	17.75	19.74	18.88	3.13	22.56
01 0039	-30	29.73	24.17	27.39	25.72	29.80
01 0040	-48	12.71	4.76	24.00	22.19	11.37
01 0041	-54	17.61	19.70	19.40	9.36	12.77

the reduction in the sum of squares from 25.95% for steepness alone to 29.32% for both variables as a pair. This is an increase of 3.37% attributable to the angle of wave approach, in contrast to its wholly negligible effect as a single variable at Hour -6 in Table 3.

In the third stage of analysis, the process elements were used in all possible combinations of three. This gives rise to four triplets as shown in Table 8. For completeness, the last column of this table also includes the three variables wave energy, wave steepness, and angle of wave approach. The greatest reduction in the sum of squares of foreshore slope is associated with the original variables wave height, angle of approach, and shore current velocity. By scanning Table 8 in comparison with Table 5, it would appear that wave height is the common thread that accounts for the strongest pairs and triplets of beach process elements. The one triplet in Table 8 that does not contain wave height still tends to show a maximum at Hour -30, although the sum of squares reduction is nearly as strong at Hour -6.

Figure 13 shows the strongest and weakest triplets from Table 8, to illustrate the manner in which three process elements combine. This may be compared with the pairs in Figure 10.

Figure 14 shows the three variables in the last column of Table 8, combining wave steepness, energy, and angle of approach. This curve suggests the diminishing influence of process elements through time, and it is tentatively accepted as an approximation to the diminishing effects of process elements on foreshore slope through time. The inset in the figure is discussed later.

TABLE 9

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS TAKEN FOUR AT A TIME

ORIGINAL VARIABLES

Project	Hour	H,T,a,V
01 0037	- 6	37.59
01 0038	-24	21.02
01 0039	-30	30.84
01 0040	-48	24.15
01 0041	-54	20 ± 2*

* Estimated from partial result.

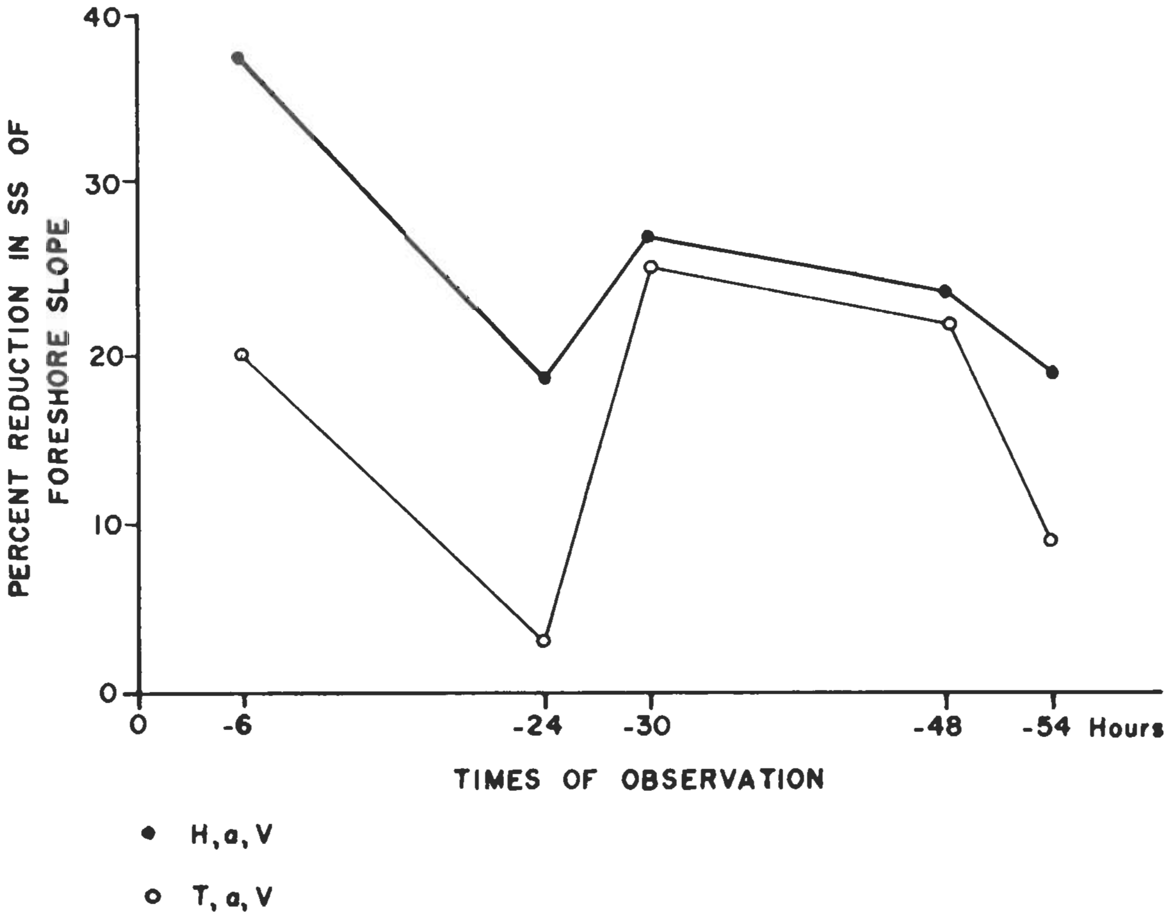

FIGURE 13

Sum-of-squares Reduction through Time for Mission Beach data,
Untransformed Variables taken Three at a Time
(See Table 9) Two Examples only.

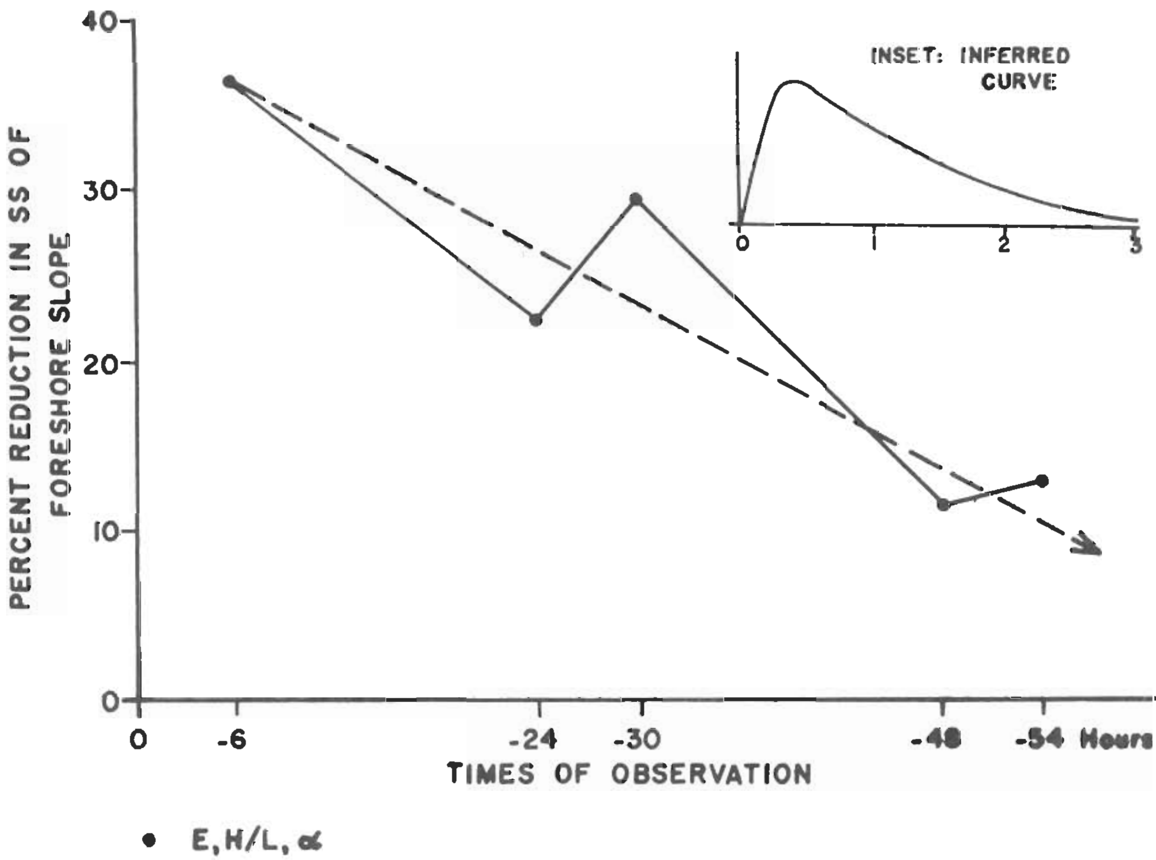

FIGURE 14

Sum-of squares Reduction through Time for Mission Beach data, Transformed Variables and Angle of Wave Approach (See Table 10). Inset shows Inferred Curve.

In the fourth and last stage of multiregression analysis all four original process elements are included simultaneously. There is one such combination, and the sum of squares reduction associated with this quadruplet is shown in Table 9. As may be noted by inspection, the successive entries yield a decreasing function through time with a secondary rise at Hour -30. Inasmuch as these are purely linear combinations of the original variables, it is thought that the curve of Figure 14, based on more meaningful physical combinations of wave steepness and energy, gives a more reasonable interpretation of the declining effect of the process elements in time.

Summary of Time-Lag Analysis. A question of considerable importance in the foregoing analysis relates to the large proportion of the total sum of squares of foreshore slope that remains unaccounted for in the multiregression analysis. As seen in Tables 8 and 9, the maximum reduction occurred at Hour -6, and the combined effect of all four original process elements reduced the sum of squares by 37.59%. The transformed variables plus the angle of wave attack effected a reduction of 36.02%. These values mean that roughly 63 or 64% of the total variability in foreshore slope remains unaccounted for by the four particular process elements used in the analysis.

Such large residuals raise numerous questions. Among others, these include the question as to whether the measurements were so approximate that measurement error dominated the results; or whether the analysis neglected other elements of such importance that only part of the story is discernible. It is the writer's belief that measurement error is important, and that some influence is exerted by the particular manner in which the variables were expressed. The absence of tidal data in the analysis is undoubtedly one major omission, and it would have been advantageous to have the slope measurements supplemented by sand samples for study of mean grain size at least.

If the present study is acceptable as a venture into methodology rather than as a definitive study of a beach under field conditions, the present reasoning may be carried further despite limitations in the present set of data.

For example, in the preceding analyses the latest time of process element measurement with respect to time of measuring foreshore slope is Hour -6. A question of some importance is whether process measurements, if made just immediately before slope measurement, say at Hour -0.5, would also show a strong effect on the measured slope. Although process element measurements were made at 3 p.m. on Zero Day, it appears (personal communication from Thorndike Saville, Jr.) that in a number of instances these observations were made after the beach profiles were surveyed, rather than just before. However, some observations preceded profiles, and purely as a possible check on interpretation, these data were analyzed as referring tentatively to "Hour Zero". The results are given in Table 10, and by comparison with earlier tables it may be seen that except for wave period and angle of approach, the reduction in the sum of squares of the slope measurements is noticeably smaller for "Hour Zero" than for Hour -6.

TABLE 10

PER CENT REDUCTION IN FORESHORE SLOPE SUM OF SQUARES ATTRIBUTABLE TO PROCESS ELEMENTS AT "HOUR ZERO"

ORIGINAL VARIABLES, PROJECT 01 0042

Variable, or Combination	Per Cent Reduction
H	12.51
T	4.36
a	7.69
V	2.55
H,T	22.48
H,a	13.89
H,V	12.58
T,a	9.54
T,V	5.33
a,V	7.72
H,T,a	22.69
H,T,V	24.37
H,a,V	13.94
T,a,V	9.55
H,T,a,V	24.39

The inference drawn from these results agrees with intuitive judgment regarding the state of the beach at some true Zero Hour (i.e., the instant when beach slope is measured). At this time the foreshore slope is an inherited characteristic, and if the particular process elements operative at Hour Zero have been essentially constant for some time, the "initial state" of the beach slope may remain unchanged. If the process elements begin to change in intensity, presumably the slope begins to adjust itself to the new conditions fairly rapidly at first, and then approaches equilibrium more slowly. The present analysis affords some insight into this process of adjustment by examining the degree to which inherited slope persists through time.

Inasmuch as the 23 days of analysis are scattered more or less randomly over several seasons, the maximum regression of slope on process elements at Hour -6 suggests that on the average through the sample of days such changes in process elements as occurred over the 54 hours of observation result in a kind of "average curve" similar to the inset shown in Figure 14. As this graph implies, process elements operating on the beach 3 days previously have had virtually all of their effects removed by later events, and that somewhere between Hour Zero and Hour -24 the process elements begin to impress their characteristics on foreshore slope as measured at Hour Zero.

The inset graph in Figure 14 shows its maximum effect somewhere between Hours -6 and -12, with a slowly declining curve becoming asymptotic to a zero effect during Day -3. If it is true that beach slope response is shaped by events of the preceding 6 or 12 hours, it would seem that a natural unit of time for studies of beaches may be a tidal cycle. This question will be examined more fully in the section on beach study design.

Relative Importance of the Process Elements. Sequential multiregression provides data for estimating the relative importance of process elements in controlling the foreshore slope, within the framework of the number of "sample days" involved, and in terms of the extent to which the several process elements changed in the sample. For the present set of data in terms of Hour -6 as an example, it is noteworthy that wave height reduced the sum of squares of foreshore slope by 32.49% as against the next largest contender, shore current velocity, with its 20.14% reduction (see Table 3). Thus wave height may be accepted as the single strongest process element when the elements are taken one at a time.

In the second stage of analysis (see Table 5) the three pairs with the greatest percentage reduction in the foreshore slope sum of squares are wave height plus shore current velocity; wave height plus angle of wave approach; and wave height plus wave period. If the percentages are taken at face value -- not fully rigorous because they are estimates in noisy data -- this suggests that shore current velocity is the second most important process element, which accords with its second-greatest value in Table 3. Addition of current velocity in the presence of wave height adds approximately 4% to the total sum of squares reduction.

In the third stage of analysis (see Table 8) the three triplets with the largest percentage reductions all have wave height in them, and two have shore current velocity, supporting the inference that these are the two most important process elements in the set. The percentage difference between wave height, angle of approach, and current velocity (H, a, V); and wave height, period, and current velocity (H, T, V), is only 0.9%, implying that wave period and angle of approach are of about the same strength as contenders for third position.

Another striking feature of the analysis is that the contribution added to wave height by wave period, angle of approach, and velocity of shore current when all four are taken together (Table 9) is only 5.10%. That is, wave height appears here to be strongly dominant as the process element of greatest importance in controlling foreshore slope. Although wave period plays a secondary role in the analysis of the original untransformed variables, it contributes significantly to the internal consistency of the data when combined with wave height to produce a variable related to wave steepness and to wave energy, as illustrated by Figures 9, 11, 12, and 14. It is perhaps also noteworthy that the combination of wave steepness, energy, and angle of wave approach, which omits the second strongest variable in the set (current velocity), reduces the total sum of squares of the foreshore slope by nearly the same amount as do all four original variables taken together.

COMMENTS ON THE ANALYSIS OF THE MISSION BEACH DATA

The Variables Involved. It is evident from the preceding section that beach studies based on measurements made in the field require very careful design in order to include all essential process elements, and to take account of at least several simultaneously-varying beach responses. The first has to do with the process elements, the second concerns a choice of dependent variables to show how the beach responds to the process elements, and the third concerns measurement of the selected variables.

These topics are discussed within the framework of the data available from Mission Beach. It is to be emphasized again that these data were collected for entirely different purposes, so that remarks regarding their limitations for the kinds of study discussed here carry no implication of similar limitations in the original water depth-measurement studies.

The present study included virtually all essential process elements except tidal influences. Shepard and LaFond (1940) have shown that tidal effects are important in sand movement along the shore, and are closely related to currents. Data on the tidal stage (mean sea level at times of observation) for Mission Beach were compiled for Zero Day, but it was anticipated that the time interval between the times of observation and the preceding spring tide would possibly be more meaningful. Circumstances, however, prevented preparation of the data for analysis, but as part of the preliminary analysis of the data the tidal stage was analyzed for Zero Hour. The percent reduction in foreshore slope sum of squares due to this element at Zero Hour was 11.21, in comparison with the 12.51% reduction due to wave height as shown in Table 10. When the tide data were added to the four variables in the last line of Table 10, the total percentage reduction was 27.51 instead of 24.39. Though this reduction is not large, the implication is that some aspect of the tidal effect may well be an important process element in terms of foreshore slope.

The selection of beach response elements as dependent variables also requires consideration. In the present study some of the unexplained variability in foreshore slope undoubtedly was related to mean particle size on the foreshore, as indicated by the interdependence of these attributes shown in Figure 1. The slope response of a foreshore may also change according to whether the foreshore is being eroded, or is being built up by the deposition of additional sand. Thus, knowledge of the bulk density or degree of packing of the sand may afford data on whether a relatively firm foreshore is being eroded during the study, or whether relatively loosely packed sand is being deposited. Presumably the slope response in these two situations may differ.

The local fluctuations in beach slope among the three traverses used in the present study suggests that measurements of beach responses should be made at more than one place on the beach. As a first approximation it would appear that study of foreshore responses should include at least two beach traverses, along each of which there would be at least two points of observation, one in the upper foreshore, and the other in the lower foreshore. Such a minimum design affords sufficient data for evaluation of local fluctuations, and for contrast of the local fluctuations with the larger-scaled phenomena of major interest.

An important aspect in all studies has to do with the manner in which the observed variables are measured, and the form in which they are expressed. Thus, in the present study the cotangent of the foreshore slope was used as the dependent variable. It is probably appropriate to consider whether the cotangent represents the "best-behaved" variable, or whether some other function of the slope angle may more effectively be used. Certainly the cotangent has immediate engineering meaning, and for this reason it was used as recorded. The direction of wave approach, expressed here as an angle, was measured clockwise from south. Inasmuch as Mission Beach is oriented essentially north-south, this means that azimuth 270 degrees represents waves coming head-on from the west. These azimuths were used directly in analyzing the data. Purely as an interpolated remark, however, it may be that an angle of approach measured in the sea area between initial refraction and final wave breaking would furnish a more meaningful value than the deep-water direction of approach.

Another aspect of the present data concerned multiple wave trains approaching the shore simultaneously. These represent different angles of approach as well as different amounts of wave energy. For purposes of the present analysis the primary wave train was used. In a specifically designed study, some manner of coping with this problem would need to be developed.

These remarks about variables used in a beach study, and of the ways in which they are expressed for analysis, are brought to the fore because it is possible unintentionally to introduce "noise" into a set of observations by neglecting important variables as well as by using measurements expressed in such a manner that they have a large "inherent variability" of their own. For example, in many studies the expression of average particle size as the logarithm of the median diameter introduces more variability into mean particle size measurements than does the use of a logarithmic mean directly.

A point of major importance in natural beach studies has to do with the accuracy and reliability of measurements made on the variables involved. It is thought that the approximate nature of the measurements of waves and currents on Mission Beach contributed importantly to the residual variability left after the analysis was completed. It is anticipated that instrumentation developed during the past decade will greatly reduce measurement error in currently designed beach studies.

Choice of Method of Analysis. The analysis of beach observations for relations among dependent and independent variables is primarily an empirical method for developing a logical ordering of beach processes and responses so that the contribution of each process on the response can be evaluated. Sequential multiregression as used on Mission Beach data is only one of several methods by which such empirical studies may be made.

A second approach, which has been widely used in similar kinds of situations, is multiple correlation analysis (Croxton and Cowden, 1955, Chapter 21). In this method all the linear correlations among all the variables in the study are examined by first computing the correlation coefficients among all possible pairs. In such analysis five variables give rise to ten correlation coefficients, four associated with the dependent variable, as shown in Table 11. As a first approximation and with some assumptions regarding normal distribution of the variables, the correlations between foreshore slope and wave height, as well as between foreshore slope and current velocity, may be considered statistically significant. Examination of the correlation coefficients among the process elements themselves suggests that wave height and current velocity are significantly correlated, and except for the marginal value of -0.377 between the angle of wave approach and wave period, the remaining interrelations among the process elements are weak. The correlation between foreshore slope and wave height in Table 11 is the single largest correlation coefficient involving the dependent variable for Hour -6. Wave height would accordingly be accepted as the single most important process element in the group, and in subsequent stages of correlation analysis this element is "held constant" statistically, during computation of the first order partial correlation coefficients. Among the first order partial correlation coefficients one is also commonly dominant. This would be considered as the second most important variable in the set, and for the third stage of analysis the computation of second-order partial correlation coefficients would involve holding both the wave height and the second most important variable constant. The process may be repeated until the contribution of all the process elements on foreshore slope can be evaluated.

TABLE 11

CORRELATION COEFFICIENTS AMONG VARIABLES
FOR HOUR -6

H	T	a	V	
-0.570	+0.024	-0.001	+0.449	Foreshore Slope
	-0.010	+0.166	-0.476	H
		-0.377	+0.081	T
			-0.072	a

(Conventional 95% level is in the range -0.45 to -1.00; and 0.45 to 1.00.)

Although multiple correlation analysis gives identically the same results as multiple linear regression when two or three process elements dominate in controlling the response, the advantages of sequential multi-regression are that all variables are examined at each stage of the analysis, rather than holding some constant for each successive stage. This is important in situations where the relative importance of process elements may change as the analysis proceeds. In the present study wave height is the single most important variable from the first stage of the analysis; and wave height and shore current velocity are the most important pair from the second stage analysis. In the third stage wave height, shore current velocity, and either the angle of approach or wave period represent the most important triplet. Thus wave height remains as a dominant variable throughout. It is not uncommon in the analysis of relatively noisy data that a variable important in one stage of analysis may be replaced by another as the analysis proceeds. Thus, by taking into account all possible combinations, this ebb and flow of the relative importance of all the beach process elements can be followed through the entire analysis.

A third general method of analysis that has been applied to situations that include a number of variables is factor analysis (Cattell, 1952). This method groups the independent variables into related sets, called factors, each of which makes some contribution to the observed response of the dependent variable.

Factor analysis is conducted by first computing all the linear correlation coefficients as in multiple correlation. These coefficients are set up in an array called the correlation matrix, from which the first factor is stripped by standard techniques, leaving a matrix of residuals that provides the basis for stripping off the second factor. When the residual matrix contains only zero values, the process of extracting factors is completed. Details of these procedures and of methods for evaluating the contribution of each factor on the response variable are described in Cattell. Factor analysis is most suitable when the number of variables is eight or more, and partly for this reason factor analysis was not applied to the Mission Beach data.

A fourth method of analysis that has been applied to beach pehnomena is trend surface analysis, which is particularly applicable to map data. Trend surface analysis is a procedure in which some mapped variable, X, is treated as a function of its geographic coordinates. The model, in terms of equation (1), is $X = f(S_1, S_2)$, where X may be either a process or response element of the general beach process. The observed value of X is divided into two components by trend analysis, such that one part is related to the systematic changes that occur on the beach, and the other is a (mathematically) random component that represents small-scaled fluctuations in the data. Miller (1956) applied this method to textural data of beach and nearshore bottom sands at La Jolla, California, and was able to show that the trend surface brought out more clearly than the original data the underlying pattern of areal distribution of particle size and sorting. In 1958 Miller and Zeigler extended the method to their model study already mentioned, as illustrated in Figure 5.

Trend surface analysis is conducted by fitting a polynomial surface to the mapped data by conventional least-squares methods. Grant (1957) has the definitive paper on the general subject, which he discusses in terms of geophysical maps. The principles, however, are independent of map scale or of the physical meaning of the measurements. Thus trend analysis provides an important method for extracting the underlying pattern of variation from a set of data in which local irregularities tend to obscure the pattern, as was emphasized by Miller (1956).

Computation of trend surfaces is described in Miller (1956) and in Krumbein (1959) for irregularly spaced observations; and by Oldham Sutherland (1955) and Grant (1957) for data arranged on grids. In trend analysis it is sometimes advantageous to examine the linear, quadratic, and perhaps higher-order surfaces in succession, as well as their deviations, in order to observe how the individual trend components add together to yield the final systematic pattern map. The mathematics are relatively straightforward though tedious, and high-speed computers are commonly used in the analysis.

Although trend analysis is commonly applied to map studies, trend surfaces can also be used to show the linear (or higher order) relations between one "dependent" variable and any two "independent" variables. For example, in terms of equation (1) a beach property G_1 or a particle property p_1 can be "mapped" on a coordinate system in which the axes are P_1 and P_2. Equation (11) on an earlier page is an example in which foreshore slope is expressed as a linear function of wave height and period. Inasmuch as one stage of multiregression includes the process factors taken two at a time, the coefficients at this stage of analysis can be used in preparation of linear surfaces for the variables concerned.

Because of the relatively high "noise level" in the Mission Beach data, this kind of trend surface analysis was not conducted in detail, although it would be an important part of any comprehensive study of beach phenomena. As an illustration, however, Figure 15 is a schematic diagram of the linear surface of foreshore slope on wave height and period for Mission Beach. Although the results of the individual sets of data varied rather widely, it appears on the average that the cotangent of the foreshore slope angle decreased with increasing wave height and increased at a lesser rate with increasing wave period, as Figure 15 shows. Inasmuch as a larger cotangent is associated with a small angle, this linear relation suggests that to a first approximation the foreshore slope is gentle for waves of small height and long period, and steep for waves of moderate height and short period. This general relation is not wholly concordant with the results of wave tank experimentation, and may be a consequence of lack of equilibrium on the natural beach. However, it would be interesting to examine these implications in terms of a designed field study.

THE DESIGN OF BEACH FIELD STUDIES

Formulation of a beach field study that seeks to relate beach reponses to process elements requires consideration of some half dozen design elements. These are summarized here in terms of the preceding discussion of beach models and from the results of analyzing the Mission Beach data. The relative importance of the design elements depends on the geologic, geophysical, or engineering objectives of the study; on the number of variables to be included; on the particular beach area or areas selected for study; and on available funds.

Studies of natural beaches vary widely in their objectives and scope, as was developed in the discussion of equation (1). Some engineering studies are related to specific beaches that may be in need or restoration or artificial nourishment. These studies have specific objectives related to the beach area in question. Others, including many geological and geophysical studies, tend to seek for relations that can be applied generally to the beach process, or that may be useful in the interpretation of the environment of deposition of ancient sedimentary rocks.

Beach studies may in fact be grouped into several classes on the basis of their objectives. The first of these includes studies in which interrelations among the characteristics of beach deposits, or of the materials that make up the beach, are emphasized. These studies do not include direct measurement of beach process elements that are operative at the time the beach properties are measured. Studies of this kind were

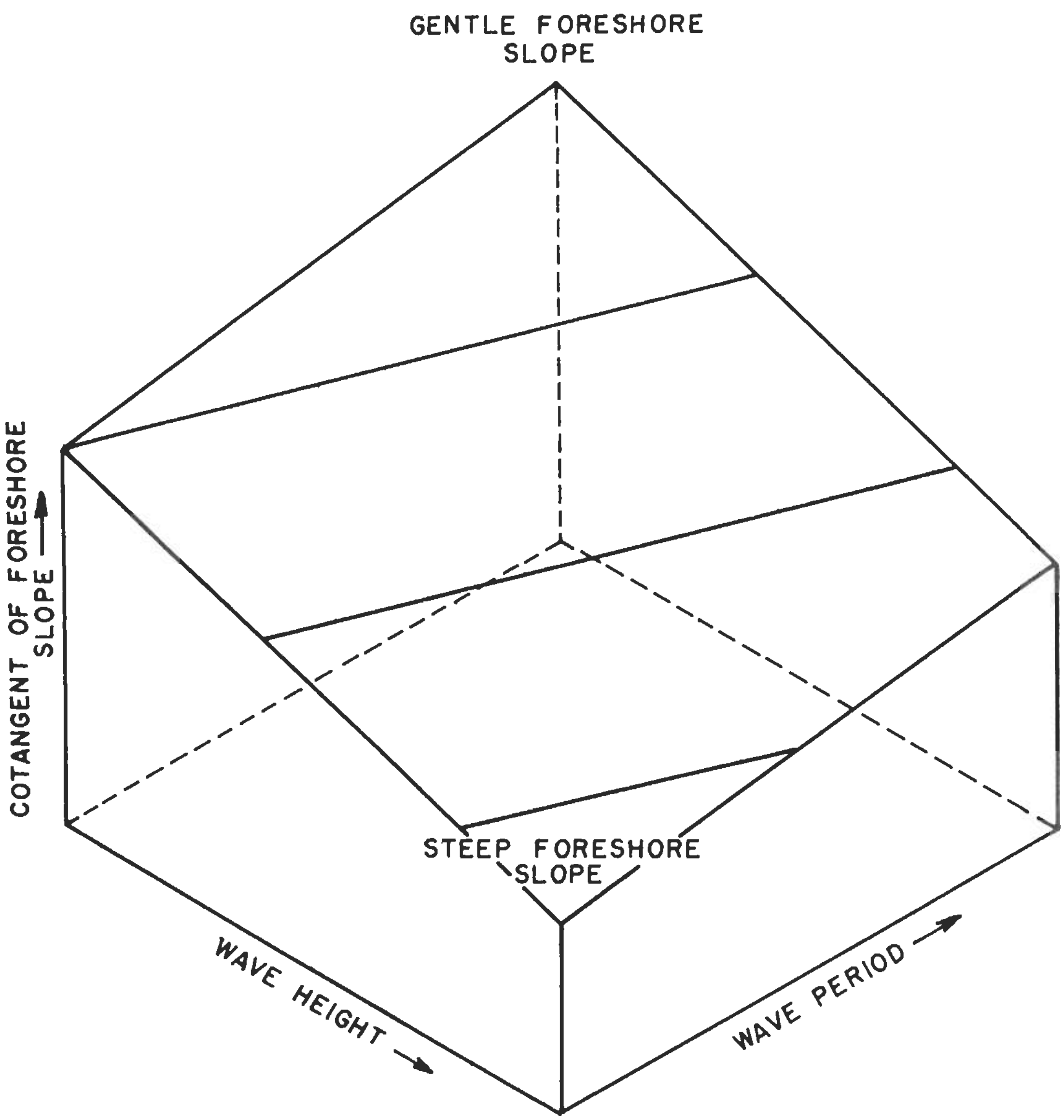

FIGURE 15

Sketch of generalized linear surface of beach slope on wave height and period over a height range of about 0.5 to 3.0 feet, and a range of period from about 6 to 20 seconds.

illustrated by Figures 1, 2, and 4. In Figure 1 the objective was to see whether some general relation is present between foreshore slope and average grain size. Figures 2 and 4 illustrate studies of changes in beach attributes as a function of distance along the beach or areally over some portion of a beach. The characteristic graphs and patterns obtained permit the inference that beach deposits have attributes that remain the same in an over-all sense, even though in detail they may differ locally during times of storm and quiet, or from season to season.

Studies of this kind, although confined to the response elements in beaches, are of considerable importance in the interpretation of ancient sedimentary rocks. The many geological papers that report on the characteristics of present day deposits is a reflection of the importance of using sediments of known origin as a guide in interpreting the conditions of formation of ancient deposits. The observation that beach deposits display certain patterns of variation and have typical geometric forms gives some assurance that when similar features are seen in ancient sediments, the inference may be made that such ancient sediments have themselves been deposited under beach conditions.

A parallel to the study of beach deposits without concurrent measurement of the process elements is illustrated by studies of beach process elements without concurrent measurement of the changing attributes of the beach deposit. In this class of beach studies are such investigations as that of breaking waves as energy release mechanisms; studies of surf patterns under different bottom-slope conditions; and studies between the angle of wave approach and the velocity of the corresponding shore current. In these geophysical studies the physical processes are of major concern, just as in geological studies the properties of beach materials are of primary interest.

It is interesting to note in passing that the study of surf patterns in relation to bottom slope investigates the dependence of a beach process element on a geometric (boundary condition) response element. In terms of equation (1), and especially in terms of the process and response models mentioned earlier, this would represent a study in which some element of the process model is studied as a function of some element in the response model, $P_1 = f(G_1)$. This designation of particular beach variables as dependent or independent indicates once more how complex and interlocked beach phenomena are as a whole.

In contrast to beach studies that emphasize either beach attributes or beach processes is a large class that examines process elements and responses simultaneously. This class of study is broader and more fundamental in that it seeks for relations between beach attributes and the processes that control them. Such combined studies have been most commonly conducted in wave tanks, but they are equally applicable to natural beaches. The study by Miller and Ziegler, mentioned earlier, is an example of such a study conducted under natural conditions.

The simultaneous study of beach process and beach response is of basic importance in beach engineering. The beach engineer is interested not only in designing structures that will withstand wave action, but also in specification of beach materials and beach slopes that will remain stable under given wave conditions. Obviously an understanding of relations between beach materials, beach slopes, and the state of the sea are fundamental in such design.

These several classes of beach field studies, though they differ in their objectives and in the aspects of the general problem that are emphasized, nevertheless do have certain features in common. All are based on measurements made on natural beaches, and hence all of them have in common a need to be structured within a physical and time framework.

With respect to the time element, it is evident from the Mission Beach data that the time lag between process and response must be taken into account in the underlying design. For practical purposes some unit of time needs to be defined during which process and response elements are to be measured, at least until measurement procedures become such that data acquisition is completely mechanized and continuous. From the discussion of Mission Beach, it is believed that one approach to the time factor can be made by using tidal intervals as basic time units. In California this would be approximately a 6-hour interval, whereas along the Atlantic coast the time interval would be of the order of 12 hours.

Consider for convenience of discussion a beach study in which response elements are confined to the foreshore, without inclusion of nearshore bottom responses. In this simplified case, measurements of the response elements are made at low tide, and observations on the process elements are made during rising and falling stages of the tide. The responses shown by the beach at each successive low tide are in part a reflection of wave and current conditions that occurred during the preceding flood and ebb in the tidal cycle, plus some "inherited" characteristics from earlier tidal cycles. Selection of the tidal cycle as the basic time unit thus provides a framework for measuring the state of the beach at each low tide, in terms of measurements of process elements that occurred during the preceding rise and fall of the tide.

The use of tidal cycles as basic units of time also provides a framework for handling the problem of the time lag between process and response elements. Thus, if R_0 represents responses measured on the beach at a given low tide, and P_0 represents the process measurements made during the immediately preceding tidal cycle, the data obtained after several cycles would match in the following way:

Low-Tide Response	Preceding Rise-Fall Cycle
R_0	P_0
R_1	P_1
R_2	P_2

where the subscripts represent matched sets of data through time. As with the Mission Beach data, response R_2 could be examined in terms of P_1, P_2, P_0, and so on backward in time. The dependence of R_2 on progressively earlier cycles of process elements could be examined by regression analysis to determine which of these most strongly affects the response R_2. This could be done on a high-speed computer as a routine procedure.

It is anticipated that the time lag between process and response may vary depending upon the incidence of storms of varying magnitude. Moreover, in periods of quiet there may be no important differences between successive response measurements, implying that the beach may approach or reach a temporary equilibrium state.

The use of such time cycles also implies a beach project that involves some minimum number of cycles. Whether a continuous set of data on some beach for a year or more is preferable to "sample sets" of data involving say ten cycles each scattered over the four seasons and perhaps on several beaches, depends partly on the specific objectives of the study, and constitutes part of the physical framework of the beach study.

As with the time element, the selection of beach areas constitutes a basic design element. If the objectives are to study a particular beach area for engineering purposes, say, then the study can be designed to supply the essential data needed for the engineering objectives. If the objectives are to generalize the results of the study to some larger class of beach phenomena, the selection of beaches involves additional considerations. Interest may focus on some particular class of beaches, such as pocket beaches between prominent headlands. For such a study, at least in the pilot stages, it would seem preferable to select a "sample" of pocket beaches so that local anomalies associated with any single beach do not cloud underlying generalizations that may apply to all pocket beaches. In contrast to pocket beaches, interest may center on straight beaches along exposed coasts, relatively free from man-made structures. Here again the optimum approach may be to select a "sample" of several short segments from several such beaches rather than to use a single long beach, in order to include a wider variety of wave, current, beach slope, and sand conditions in the project.

The particular beaches selected for study may be chosen because they display certain characteristics of interest or because they are readily accessible. In contrast to this approach, the selection may be made by some randomization procedure from a "population" of beaches or of beach segments. Purposive selection of a beach for certain attributes implies use of a fixed model of the kind earlier described as an analytic model, whereas introduction of a randomization procedure is more appropriately related to a statistical model.

A third major element in beach study design has to do with the variables to be measured, and the construction of a specific model for the study. This part of design is related directly to the specific objectives of the study, but some general remarks are appropriate. The process elements that appear to be basic in any study include wave height, wave period, and angle of approach; as well as direction and velocity of shore currents. On the response side it seems desirable to include at least one geometric attribute of the beach, such as foreshore slope, and at least two particle properties, such as average grain size and degree of sorting.

A fourth general element of design concerns the times of observation of process and response elements, as well as the duration of the study. If the tidal cycle is used as a unit of time, measurement of the response elements would occur at low tide, or could be made progressively across the foreshore as the tide recedes, inasmuch as any one set of response measurements is matched with that of the preceding tidal cycle. Response measurements would be made at more than one point on the beach, both because of the likelihood that some locally anomalous feature may characterize that point, plus the advantage of having several points of observation from which the amount of local variability could be estimated.

Measurement of process elements poses a more difficult problem, in that wave and current conditions may change markedly during a single tidal cycle. Unless continuous wave and current measurements can be made, to provide a basis for noting when important changes occur, some form of sampling is required. One approach may be to make observations for say 15 minutes at the rising half tide, at full tide, and at the falling half tide. An alternative procedure may be to select times of observation by some randomized sampling process, or to set up a series of standard times each day, to obtain data covering all of the tidal stages through a successive series of tidal cycles.

Selection of a model for a particular study brings up several topics touched upon earlier. The point of view here is that the model should contain both process and response elements, though it is recognized that many more geological studies can profitably be made on response elements alone; and that further understanding of process elements can be advanced by models emphasizing the process side of beach phenomena. Whatever the underlying objectives, it would appear from earlier discussion that a statistical model that takes explicit account of the fact that observational data, both process and response, are sample data, would be preferable to an "analytic" model that accepts observations at their face value for direct use in deriving generalizations.

This last statement is in large part arbitrary, inasmuch as some detailed beach studies may in fact take into account each wave and every increment of beach material during a given time of observation. Nevertheless, for studies whose objectives are to investigate relations between process and response elements for possible application to engineering, the statistical model has advantages in that it provides a basis for stating the likelihood that generalizations or predictions are "true" within some specified limits. It is this rigorous combination of statistical analysis and probability theory that gives statistical models their advantages for many field studies.

The distinction between the statistical and non-statistical approaches can be made clear by an example. Suppose that foreshore slope and average grain size are to be studied in terms of wave and current characteristics. Observations are made on the variables involved, and in the "analytic" case these observations are analyzed by least-squares methods to determine the underlying systematic relations among the variables. Deviations from these systematic effects are treated as "noise" that may cloud the underlying relations, and in extreme cases the noise may be ignored as representing those accidental non-systematic fluctuations that are almost always present in natural data.

In the corresponding statistical model the same kinds of observations are made (but with some consideration for sampling procedures), and the same kinds of least-squares computations are conducted. The "noise", however, is given as much consideration as the systematic effects in the data, inasmuch as this "unexplained" residual variability can be used as a measure of the uncertainty (or, in a more positive sense, the reliability) of statistical inferences drawn from the data. Moreover, study of the residual variability commonly discloses that a large part of it may be related to small-scale fluctuations in process or response elements; and that the true residual, in the sense that it represents completely unpredictable random effects, may be very small.

As a general summary of this section on design features in beach studies, it is evident that the framework of a study is very largely determined by the objectives of the study. Once these are decided upon, the selection of beach areas, of variables to be measured, and the formulation of a beach model, are also largely determined. The model normally specifies the manner of taking and analyzing observational data for making inferences, predictions, generalizations, or decisions in terms of the original objectives. The part played by the model in this scheme is mainly one of structuring the data.

It is also to be emphasized that the designer has complete freedom in defining the scope and objectives of his beach study, usually within a framework of available time and funds. If a statistical model is decided upon, the designer also has complete freedom in defining the conceptual populations in which he is interested, and from which samples are to be drawn. This population framework of beach studies requires a separate paper for adequate treatment, but some discussion of the subject can be found in Miller (1956) and in Krumbein (1960).

Statistical models invariably include some elements of randomization, such as are associated with sampling, for example, in order to remove personal bias from the study and to assure the validity of certain kinds of statistical inferences. In the "analytical" model, these randomization procedures may be ignored, but this places a heavier responsibility on the designer to demonstrate that his generalizations are validly supported by his data.

REFERENCES

Beach Erosion Board, (1933) Interim report: Office of the Chief of Engineers, U. S. Army Washington, D. C.

Cattell, R. B. (1952) Factor Analysis, Harper and Brothers, New York.

Chamberlin, T. C. (1897). The method of multiple working hypotheses: Jour. Geology, vol. 5, pp. 837 ff.

Grant, F. (1957) A problem in the analysis of geophysical data: Geophysics, vol. 22, pp. 309-344.

Johnson, D. W. (1919) Shore processes and shoreline development: John Wiley and Sons, New York.

Krumbein, W. C., and Miller, R. L. (1953) Design of experiments for statistical analysis of geological data: Jour. Geology, vol. 61, pp. 510-532.

Krumbein, W. C. (1959) Trend surface analysis of contour-type maps with irregular control-point spacing: Jour. Geophysical Research, vol. 64, pp. 823-834.

Krumbein, W. C. (1959a) The "sorting out" of geological variables illustrated by regression analysis of factors controlling beach firmness: Jour. Sed. Petrology, vol 29, pp. 575-587.

Krumbein, W. C. (1960) The "geological population" as a framework for analyzing numerical data in geology: Liverpool and Manchester Geol. Jour., vol. 2, pp. 341-368.

Miller, R. L.(1956) Trend surfaces: Their application to analysis and description of environments of sedimentation: Jour. Geology, vol. 64, pp. 425-446.

Miller, R. L., and Zeigler, J. M. (1958) A model relating dynamics and sediment pattern in equilibrium in the region of shoaling waves, breaker zone, and foreshore: Jour. Geology, vol. 66, pp. 417-441.

Oldham, C. H. G., and Sutherland, D. B. (1955) Orthogonal polynomials; their use in estimating the regional effect: Geophysics, vol. 20, pp. 295-306.

Pelto, C. R. (1954) Mapping of multicomponent systems: Jour. Geology, vol. 62, pp. 501-511.

Pettijohn, F. J. and Lundahl, A. C. (1943) Shape and roundness of Lake Erie Beach sands, Jour. Sed. Petrology, vol. 13, pp. 69-78.

Saville, Thorndike Jr. and Caldwell, Joseph M. (1953) Accuracy of hydrographic surveying in and near the surf zone. Technical Memorandum No. 32. Beach Erosion Board, U. S. Army Corps of Engineers.

Shepard, Francis P. and LaFond, Eugene C. (1940) Sand Movements along the Scripps Institution pier. American Journal of Science, vol. 238, pp. 272-285.

TABLE A

MISSION BEACH DATA, HOUR -6

(Project 01 0037)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent ** Foreshore Slope at Hour Zero
1	1.5	12	260	0	28.7
2	1.5	12	245	-20	28.4
3	1.5	18	180	-45	25.9
4	1.0	15	240	-10	28.2
5	1.5	14	265	-35	32.7
6	1.0	15	250	-60	41.9
7	0.5	15	255	76	44.0
8	1.5	17	260	-30	33.8
9	0.5	10	245	0	37.4
10	2.0	15	250	35	32.3
11	0.7	13	250	-10	34.4
12	1.0	14	240	5	34.9
13	0.3	17	220	35	38.4
14	1.0	15	180	30	40.6
15	1.0	14	180	-10	44.2
16	0.5	18	220	7	42.8
17	1.0	15	230	20	38.8
18	0.5	17	235	50	48.0
19	1.0	14	240	0	38.3
20	0.5	12	250	-10	47.8
21	1.5	14	270	-14	50.1
22	1.5	15	240	-49	30.5
23	1.5	15	240	-60	26.0

* V is positive if northward-flowing; negative if southward flowing.

** These values are the same in each table; they represent the average of the slope values measured at Zero Hour along the three ranges shown in Fig. 6.

TABLE B

MISSION BEACH DATA, HOUR -24
(Project 01 0038)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent Foreshore Slope at Hour Zero
1	1.5	9	265	15	28.7
2	1.0	11	250	-10	28.4
3	3.0	10	285	-20	25.9
4	1.0	16	250	60	28.2
5	1.0	20	195	20	32.7
6	1.0	13	240	-51	41.9
7	1.0	12	255	75	44.0
8	1.5	15	245	0	33.8
9	1.5	15	245	40	37.4
10	2.0	13	230	0	32.3
11	0.5	7	250	0	34.4
12	1.0	14	240	10	34.9
13	0.5	13	220	5	38.4
14	1.0	17	230	65	40.6
15	1.5	13	265	0	44.2
16	1.0	15	220	35	42.8
17	1.5	11	230	0	38.8
18	0.5	16	235	10	48.0
19	1.0	13	235	20	38.3
20	0.5	12	250	0	47.8
21	1.5	12	245	-70	50.1
22	1.0	11	240	10	30.5
23	1.5	15	240	-35	26.0

*V is positive if northward-flowing; negative if southward-flowing.

TABLE C

MISSION BEACH DATA, HOUR -30

(Project 01 0039)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent Foreshore Slope at Hour Zero
1	1.0	12	265	10	28.7
2	2.0	13	280	40	28.4
3	2.8	8	270	-30	25.9
4	0.5	8	255	30	28.2
5	1.0	15	270	10	32.7
6	0.5	11	250	-30	41.9
7	0.5	12	245	-40	44.0
8	1.0	15	245	-48	33.8
9	0.8	14	255	0	37.4
10	1.0	10	250	20	32.3
11	1.0	10	255	0	34.4
12	1.5	13	245	-31	34.9
13	0.5	13	220	- 5	38.4
14	0.5	13	230	0	40.6
15	1.5	17	180	10	44.2
16	0.8	16	220	0	42.8
17	1.0	11	230	5	38.8
18	1.0	14	235	- 5	48.0
19	1.0	14	230	55	38.3
20	0.5	12	255	- 5	47.8
21	2.0	14	250	10	50.1
22	1.0	12	240	-72	30.5
23	1.5	15	240	-49	26.0

* V is positive if northward-flowing; negative if southward-flowing.

TABLE D

MISSION BEACH DATA, HOUR -48

(Project 01 0040)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent Foreshore Slope at Hour Zero
1	2.0	13	265	0	28.7
2	1.0	15	280	-10	28.4
3	1.0	12	265	-50	25.9
4	1.0	15	260	-90	28.2
5	2.5	14	255	-30	32.7
6	0.5	13	245	0	41.9
7	1.0	11	250	-90	44.0
8	2.0	7	260	-85	33.8
9	0.5	6	250	-20	37.4
10	1.5	17	225	-15	32.3
11	1.0	12	260	40	34.4
12	1.0	15	240	72	34.9
13	0.5	12	220	50	38.4
14	0.5	17	225	20	40.6
15	1.5	12	245	- 5	44.2
16	1.0	15	220	5	42.8
17	1.5	13	230	75	38.8
18	1.5	13	235	25	48.0
19	0.5	15	265	20	38.3
20	0.5	12	260	-75	47.8
21	1.5	15	255	-123	50.1
22	0.8	14	240	30	30.5
23	1.0	11	240	10	26.0

* V is positive if northward-flowing; negative if southward-flowing.

TABLE E

MISSION BEACH DATA, HOUR -54

(Project 01 0041)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent Foreshore Slope at Hour Zero
1	2.0	11	265	0	28.7
2	2.0	10	280	30	28.4
3	1.5	14	270	-60	25.9
4	1.0	16	190	-10	28.2
5	2.0	14	260	-20	32.7
6	1.0	13	245	-27	41.9
7	0.5	11	250	-12	44.0
8	1.5	6	285	-15	33.8
9	1.5	7	255	0	37.4
10	1.0	10	240	0	32.3
11	2.0	14	260	0	34.4
12	1.0	15	240	-15	34.9
13	0.5	14	220	15	38.4
14	0.5	16	225	48	40.6
15	1.5	18	180	20	44.2
16	0.5	16	220	7	42.8
17	1.0	15	220	35	38.8
18	1.0	13	235	15	48.0
19	1.5	14	230	15	38.3
20	0.5	13	250	20	47.8
21	1.5	13	255	-59	50.1
22	1.0	14	240	30	30.5
23	1.0	12	240	-77	26.0

* V is positive if northward-flowing; negative if southward-flowing.

TABLE F

MISSION BEACH DATA, HOUR ZERO**

(Project 01 0042)

Day Code	H, ft.	T, sec.	a, degrees	V, ft./min.*	Cotangent Foreshore Slope at Hour Zero
1	2.0	15	290	-30	28.7
2	1.0	10	270	0	28.4
3	1.5	13	280	-35	25.9
4	1.0	15	240	- 5	28.2
5	1.0	15	245	-15	32.7
6	0.5	12	255	-60	41.9
7	0.5	14	180	45	44.0
8	1.0	17	225	50	33.8
9	0.5	15	190	30	37.4
10	0.5	13	255	20	32.3
11	1.0	12	250	5	34.4
12	1.5	15	240	15	34.9
13	0.3	15	245	55	38.4
14	1.5	16	265	0	40.6
15	1.5	16	250	3	44.2
16	1.0	16	220	0	42.8
17	1.0	14	230	0	38.8
18	1.0	16	235	-15	48.0
19	1.0	14	245	0	38.3
20	0.5	12	250	-25	47.8
21	1.5	15	270	10	50.1
22	1.5	15	240	-35	30.5
23	2.0	14	245	-20	26.0

* V is positive if northward-flowing; negative if southward-flowing.

** As mentioned in the text, Hour Zero refers to time of slope measurment; the process elements were in part measured later.

TABLE G

LIST OF OBSERVATION DATES ON FORESHORE SLOPE *

February 8, 1950
February 20, 1950
May 5, 1950
May 12, 1950
May 18, 1950
May 26, 1950
June 2, 1950
June 9, 1950
June 16, 1950
June 23, 1950
June 30, 1950
July 7, 1950
July 21, 1950
August 4, 1950
August 11, 1950
August 18, 1950
August 25, 1950
September 1, 1950
September 8, 1950
September 15, 1950
December 12, 1950
April 24, 1951
April 25, 1951

* These are Zero Days; for February 8, 1950, Day -1 is February 7, and Day -2 is February 6.

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